Dominant Currency Debt *

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Abstract

We propose a "debt view" to explain the dominant international role of the dollar and provide broad empirical support for it. Within a simple capital structure model in which firms optimally choose the currency composition of their debt, we derive conditions under which all firms issue debt in a single, "dominant" currency. Theoretically, it is the currency that (1) depreciates in global downturns over horizons of typical debt maturity of firms and (2) has the steepest nominal yield curve. Both forward-looking and historical measures suggest that the dollar fits this description better than all major currencies. The debt view can jointly explain the fall and the rise of the dollar in international debt markets over the last two decades. It also offers insights into the future of the dominance of the dollar in the aftermath of the Covid-19 crisis.

Keywords: Dollar debt, dominant currency, exchange rates, inflation

JEL Classification Numbers: E44, E52, F33, F34, F41, F42, F44, G01, G15, G32

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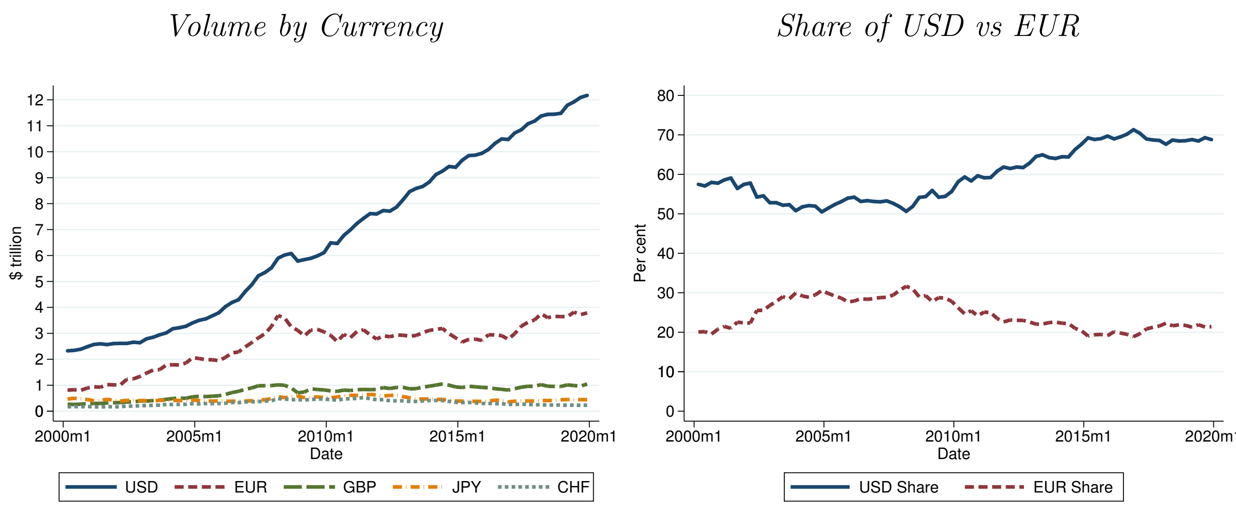
The dollar is the most common currency of choice for debt contracts worldwide. According to the Bank for International Settlements, the dollar-denominated credit to non-banks outside the United States amounts to around $12 trillion. While the dominance of the dollar had declined before 2008, the dollar has strengthened its international role since the Global Financial Crisis (Figure 1).[1]

Figure 1: Currency Denomination of International Debt and Cross-Border Borrowings of Non-Banks (Amounts Outstanding)



Source: Bank for International Settlements

In this paper, we study how a single currency can become the most common currency of choice for denominating debt contracts, i.e. *the dominant currency*, why that choice is the dollar, and why the dominance of the dollar may have declined and recovered in the last two decades. To fix ideas, in this paper our focus is not to answer why emerging market firms issue debt in dollars as opposed to local currency. Instead, our primary focus is on why large, global firms issue debt in dollars as opposed to other major safe haven currencies, such as the euro or the yen.

According to the conventional view, debt issuance in dollars is investor-driven. Investors

[1]Similar patterns were also previously documented (see, for example, ECB (2017) and Maggiori, Neiman and Schreger (2019)).

prefer holding safe assets that tend to appreciate in bad times. Therefore, firms choose currency denomination of their debt to cater to investors' demand. There are three potential challenges to this view. First, we show empirically in this paper that the dollar is not the "safest" among the major currencies, such as the euro, the Japanese yen or the Swiss franc. Second, nominal interest rates in dollars are higher than those in these other major currencies. Third, the dollar increased its international role after the Bretton Woods, even as it depreciated considerably against other major currencies in the 1970s (Gourinchas (2019)).

We propose the debt view, in which debt issuance in dollars is borrower-driven. In the baseline version of our model, firms finance themselves by issuing equity and nominal, defaultable debt to optimize the trade-off between tax benefits of debt and the risk of default.[2] Debt can potentially be issued in any currency. Firms issue in dollars if dollar debt maximizes this trade-off. Our first theoretical result is that, independent of the investors' preferences, firms always issue debt in the most "CAPM-risky" currency. It is the currency that, controlling for issuance costs, has the highest covariance with the stock market *over the horizons of debt maturity.* We call it the "dominant" currency. If investors' marginal utility co-moves negatively with the stock market, such debt is unattractive for debt-holders, and yet firms still prefer issuing in this currency.

These features of the debt view have two implications and can explain the challenges to the conventional view outlined above. First, dollar debt represents a better hedge for firms against economic downturns than other major currencies, such as the euro, the yen or the Swiss franc, making it easier to repay at times of distress. Second, the currency in which firms prefer issuing debt should have a higher risk premium. As a result, the dollar is the dominant currency for denominating debt, not despite being the riskiest of the major currencies, but precisely because of it. A higher associated risk premium leads to higher

[2] We think of our model as being best applicable to large international firms deciding whether to issue debt in one of the major international currencies with comparable market liquidity and issuance costs. Nikolov, Schmid and Steri (2018) show that the trade-off theory efficiently explains capital structure dynamics for large firms. By contrast, other theories need to be developed for smaller firms facing a high degree of informational asymmetry.

nominal dollar interest rates. Finally, we show that since the end of Bretton Woods, stock market declines tended to be followed by the depreciation of the dollar, incentivizing firms to borrow in dollars cementing its dominant international role.

Empirically, we test our prediction that the dollar is the "CAPM-riskiest" among the major international currencies. The first prediction of our model is that the dominant role of the dollar in international debt markets might be attributed to the expectations of market participants of a positive co-movement of the dollar with the stock market *over the horizons of debt maturity of firms*, which is typically around five years.[3] We show empirically that this is indeed the case in two ways. First, we use asset-price implied forward-looking expectations of market participants regarding the covariance of the EUR/USD exchange rate and the S&P 500, directly computed from so-called quanto contracts. Second, we compute the historical covariances between the dollar and global stock markets.

A direct way of computing the forward-looking covariance between the stock market and exchange rates is by using so-called quanto forward contracts (Kremens and Martin, 2019). A euro-quanto forward contract for S&P 500, for example, pays off the level of the S&P 500 index *in euros* when the contract matures. As opposed to a contract that pays off the S&P 500 in dollars, the value of this contract depends on the anticipated covariance between the index and the EUR/USD exchange rate. Hence, the price of this contract reflects the expectations of investors about currency returns and currency risk premia. Kremens and Martin (2019) compute the quanto-implied covariance for contracts with a two-year maturity and find that the quanto-implied covariance of the EUR/USD exchange rate with S&P500 exhibited a robust downward trend in the post-crisis period and has become negative in the recent years. A negative quanto-implied risk premium (QRP) means that market participants believe that the euro will *appreciate* against the dollar when the S&P 500 falls, in line with our theory for why firms would issue debt in dollars.

Our theoretical characterization of the dominant currency also has direct implications for

[3] See section 4 and also Cortina, Didier and Schmukler (2018) for typical debt maturities.

the time-series dynamics of the shares of the dollar- and euro-denominated debt. Namely, keeping the distribution of issuance costs constant across firms, our model predicts the share of dollar-denominated debt relative to that of euro-denominated debt is negatively related to the QRP. Consistent with the predictions of our model, we find a strong negative relationship between quanto-implied covariances and the share of dollar debt, suggesting that forward-looking expectations of currency returns are an important driver of firms' debt currency denomination choice. Moreover, we interpret this fact as strong evidence of a distinctive prediction of our theory, that is, changes to the currency composition of debt can occur in high frequency and are related to forward-looking expectations since our regressions are at a quarterly frequency.

The debt view also assigns an important role for monetary policy if relative inflation between two countries is an important driver of exchange rates.[4] Similar to the predictions regarding the QRP, our model implies that the share of dollar-denominated debt relative to that denominated in euro is positively related to the inflation risk premium of the dollar and negatively related to that of the euro. Therefore, according to our model, there is a close link between central bank inflation stabilization policies in global downturns and firms' debt currency choice. Strikingly, we find that debt currency shares move more tightly with the expectations about inflation risk premia, especially in the Eurozone, and that explains debt issuance patterns over the last two decades, suggesting that deflation risk in the Eurozone after the crisis is a possible reason for why the euro lost its momentum.[5]

Computing covariances from historical data, we find that the dollar co-moves positively with the stock market at horizons that typically accord with the debt maturity of firms. This pattern does not contradict the well-documented tendency of the dollar to appreciate

[4]See, for example, Imbs, Mumtaz, Ravn and Rey (2005), Chowdhry, Roll and Xia (2005) for evidence in favor of the relative PPP, as well as Chernov and Creal (forthcoming) who argue that PPP is an important driver of long-horizon currency risk premia.

[5]The importance of accommodative monetary policy in helping reduce real debt burdens of firms and the differences across central banks in accomplishing this goal is also acknowledged by the European Central Bank (ECB). See, for example, Praet (2016) and Cœuré (2019).

in bad times over shorter horizons (see, for example, Gourinchas, Govillot and Rey (2017)). We, indeed, find that the dollar co-moves negatively with the stock market for horizons up to a year, but the sign of the covariance switches for longer horizons corresponding to typical maturities of debt firms issue.

To gain a deeper understanding of the term structure of dollar-stock market co-movement, we further decompose the long-horizon covariances into shorter-term contemporaneous and lead-lag relationships between the dollar and the stock market, proxied by S&P 500 or the MSCI World Index. While the contemporaneous covariance is negative, we find that the stock market positively predicts the dollar, and the strength of this effect is sufficient to produce a sign change at longer horizons. To formally estimate the lead-lag relationships between the dollar and the stock market, we estimate a VAR for their joint return process and study the impulse response for their joint dynamics after a stock market depreciation shock. We find that, after such a shock, the dollar significantly depreciates in future periods, consistent with our theory.

As the dollar co-movement with the stock market increases over longer horizons, our model predicts that the propensity to issue dollar-denominated debt increases with debt maturity. We use granular bond issuance data to test this prediction and find strong support for it.

The sign change in the term structure of the dollar-stock market covariance (negative for short horizons, positive for long horizons) naturally raises the question: what would happen if the firm could rebalance its capital structure after a short run dollar appreciation? To answer this question, we solve a dynamic version of our model with an intermediate period at which the firm can issue more debt or buy back debt. Our first observation is that independent of the underlying shock dynamics, the firm never buys back debt because it would mean foregoing the tax benefits. This is a version of the leverage ratchet effect of Admati, Demarzo, Hellwig and Pfleiderer (2018). Thus, when the firm issues long-term debt at time $t = 0$, it anticipates holding this debt all the way until expiry, making this part of the debt choice problem effectively static. The next question is: Does the possibility of issuing more debt at the intermediate period alter the currency composition of debt ex-ante? Despite the extreme complexity of the dynamic capital structure problem, we derive the optimal debt issuance policy in closed form and uncover a novel *inter-temporal trade-off mechanism*. According to this mechanism, the firm chooses between receiving the tax benefits today and paying the (effective) cost of default tomorrow.

We characterize this novel trade-off explicitly in terms of the slopes of nominal yield curves across currencies and show that the firm always selects the currency with the steepest yield curve. The dollar also fits this description compared to other major currencies. In addition, we test a prediction of our model that the fraction of dollar debt issuance co-moves positively with the difference in yield curve slopes between the dollar and the euro (using the Treasury yield curves in the United States and Germany). Consistent with our theory, we find that, quite remarkably, regressing dollar debt issuance on this yield slope differential produces statistically significant estimates with the predicted sign.

Finally, the Covid-19 crisis and ensuing developments in exchange rate markets shed further light on our theory. Our theory suggests that borrowers would prefer to issue debt in dollars if they expect it to depreciate in response to a global shock against other major currencies, in part due to easing by monetary policy. Following the Covid-19 shock, the Federal Reserve promptly and forcefully eased monetary policy and rolled out crisis programs. As a result, the dollar depreciated against all major international currencies. Unlike previous episodes, this depreciation happened almost immediately and to the order of around 10% in a few months. The depreciation of the dollar provided a material hedge for firms with nominal dollar debt as opposed to debt in euros, yen, or the Swiss francs. Moreover, inflation risk premia for the dollar increased compared to the euro. In this context, our theory predicts higher dollar debt issuance in the post-Covid period. We show evidence using granular issuance data that firms tended to issue more dollar debt compared to other major currencies in the aftermath of the Covid-19 crisis, controlling for other factors and firm fixed effects. The nature of the shock, the depreciation of the dollar, the impact of monetary policy, and the debt issuance patterns in the aftermath of the Covid-19 crisis are all in line with the mechanisms discussed in our paper.

Related literature. The international role of the dollar has received a lot of attention in the recent literature. The dollar is omnipresent in all parts of the global financial system (CGFS (2020), Gopinath and Stein (forthcoming) and Gourinchas, Rey and Sauzet (2019)). This includes international trade invoicing (see Goldberg and Tille (2008), Gopinath (2015), Casas, Díez, Gopinath and Gourinchas (2017)); global banking (Shin (2012), Ivashina, Scharfstein and Stein (2015), Aldasoro, Ehlers and Eren (2019)); corporate borrowing (Bruno, Kim and Shin (2018), Bruno and Shin (2017), and Giovanni, Kalemli-Ozcan, Ulu and Baskaya (2017)); central bank reserve holdings (Bocola and Lorenzoni (2020) and Ilzetzki, Reinhart and Rogoff (2019)); and global portfolios (Maggiori, Neiman and Schreger (2019)). Our paper adds to the growing literature that studies the international role of the dollar.[6]

Our main contribution is the introduction of the "debt view" in explaining the international role of the dollar. Current explanations can be broadly classified into three categories. First is the "trade view," wherein trade invoicing in dollars is the reason for the dollar's role in the global economy (see, for example, Gopinath and Stein (forthcoming)). Second is the "safe asset view," in which the dollar is dominant because of its safe haven properties (see, for example, He, Krishnamurthy and Milbradt (2019), Farhi and Maggiori (2018), and Jiang, Krishnamurthy and Lustig (2018)) and the global demand for safe assets (Caballero,

[6]See also, for example, Matsuyama, Kiyotaki and Matsui (1993), Rey (2001), Caballero and Krishnamurthy (2002), Caballero, Farhi and Gourinchas (2008), Caballero and Krishnamurthy (2009), Devereux and Shi (2013), Chahrour and Valchev (2017), Mukhin (2018), Farhi and Maggiori (2018), He, Krishnamurthy and Milbradt (2019), Drenik, Kirpalani and Perez (2019) and Bahaj and Reis (2020a).

Farhi and Gourinchas (2008), Caballero, Farhi and Gourinchas (2015), Caballero, Farhi and Gourinchas (2017)). Third is the "vehicle currency view," wherein the dominance of the dollar arises from its international use as a vehicle currency (see for example Goldberg and Tille (2008)).

The debt view of the dollar's dominance assigns an important role in the choice of the debt currency denomination of firms, driven by *forward-looking expectations about exchange rates and monetary policy.*[7] The debt view focuses on the medium run to account for typical debt maturity of firms, and in that complements other theories which focus on the short run frictions such as price stickiness, or the short-run appreciation of the dollar in bad times as insurance to investors. In contrast to other theories, we show that a dominant currency equilibrium in the debt market can arise without relying on network effects, price stickiness, pricing complementarities, and safety demand.

Three closely related papers to ours are by Gopinath and Stein (forthcoming), Jiang, Krishnamurthy and Lustig (forthcoming), Liao (2020) and Bahaj and Reis (2020b). Gopinath and Stein (forthcoming) demonstrate how the dollar can emerge as a key international currency starting from its role in trade invoicing and in turn affecting global banking, which in turn affects currency denomination of bank deposits and firm borrowing endogenously. While their main focus is emerging markets and bank-intermediated debt, our results apply mostly for the currency choice of large, global firms and also apply to market-based financing, and the dollar's dominant role arises due to its risk properties. Jiang, Krishnamurthy and Lustig (forthcoming) find that investors attach a convenience yield for dollar safe assets that can be observed from covered interested parity deviations. An implication of their results is that firms would issue dollar debt to reap the benefits of this convenience yield. Liao (2020) shows that firms issuance flows or euro and dollar debt between the Eurozone and the United States respond to covered interest parity deviations. Bahaj and Reis (2020b)

[7] The main mechanism in our model is similar to the one in Gomes, Jermann and Schmid (2016); however, in an international setting.

also assign an important role for monetary policy for the internationalization of currencies as in our paper, studying the Chinese renminbi's jumpstart as an international currency. Our results complement the findings of these papers. We find that firms choose the dollar as opposed to other currencies regardless of the preferences of investors, and issuance of dollar debt could be determined by its favorable risk properties compared to other major currencies, such as the euro, the Japanese yen, or the Swiss franc.

A growing literature studies reasons for persistent uncovered interest rate parity (UIP) violations and its implications for the choice between local and foreign currency borrowing of emerging market firms (see, for example, Baskaya, di Giovanni, Kalemli-Ozcan and Ulu (2017) and Salomao and Varela (2020)). Our primary interest lies in understanding why it is the dollar that is the "dominant foreign currency" as opposed to similar currencies with similarly deep and liquid markets, such as the euro. Thus, we abstract from many features of currencies that lead to UIP deviations, in reality, to highlight our main mechanism, even though it would be possible to capture these in reduced form through differential issuance costs. In the internet appendix E, we provide some theoretical and empirical results for the dominant and local currency mix in the debt of a cross-section of emerging market firms that depend on the properties of local inflation and its relation to the US inflation.

Our paper is also related to the extensive literature on long-term nominal debt and its real effects, including debt deflation (Fisher (1933)), debt overhang (Myers (1977)), and leverage dynamics (Gomes, Jermann and Schmid (2016)).

1 Theory

In this section, we build a simple model that lays out conditions for the currency choice of firms' debt issuance. Our main theoretical result is that firms' currency choice boils down to a simple statistic: the covariance between stock returns and exchange rate returns. This covariance governs the choice of debt currency, regardless of the stochastic discount factor of lenders. In the first part, we take exchange rates as given. In the second part, we add slightly more structure to exchange rate determination. This allows us to assign a role for inflation and monetary policy in determining firms' currency choice.[8]

1.1 Model

Time is discrete, indexed by $t = 0, 1, \cdots$. A large, international firm is infinitely lived and generates after tax cash flows of $\Omega_t Z_t$ where Ω_t is the common productivity shock, which is measured in dollars, and Z_t is an idiosyncratic shock. If the firm generates cash flows in different currencies, we just multiply them with the respective exchange rates and then aggregate them to get the total dollar cash flows. We assume that Z_t follows a geometric random walk, $Z_{t+1} = Y_{t+1} Z_t$ where Y_t are i.i.d. and have a density $P(Y_{t+1} = y) = \ell y^{\ell-1}$, $y \in [0, 1]$ and $\ell > 0$. This assumption is common in the literature on international trade (see, for example, Melitz (2003)) and is made for tractability. We denote by $\Phi(y) \equiv P(Y \leq y) = y^\ell$ the cumulative distribution function of idiosyncratic shocks. All cash flows are priced with a common, exogenously given *dollar* stochastic discount factor $M_{t,t+1} = M^{\$}_{t,t+1}$.[9]

Firms finance themselves by issuing both equity and defaultable nominal bonds in *any* of the N currencies, maturing in one time period.[10] Each bond has a nominal face value of one currency unit, and the firm is required to pay a coupon of c currency units per unit of outstanding debt.[11] We denote by $B_{j,t}$ the stock of outstanding nominal debt at time t

[8] While we take a partial equilibrium approach in this paper for simplicity, in an earlier working paper version, we solved the model in general equilibrium (Eren and Malamud (2019)). All results and intuitions go through in a general equilibrium setting as well.

[9] The choice of the dollar as the reference currency is made purely for tractability. See Lemma A.2 where we re-derive all expressions in the domestic currency.

[10] We interpret this one single period as the typical maturity of debt of the order of several years. See, for example, Cortina, Didier and Schmukler (2018). It is known that the dollar tends to appreciate over the short term during crises (see, for example, Maggiori (2017) and Farhi and Maggiori (2018)). So some portion of the long-term debt may become due exactly during a crisis. We abstract from such considerations. However, that said, firms may keep dollar cash buffers to mitigate potential problems from the dollar's short-term risk profile.

[11] Apart from the multiple currencies assumption, when modeling the financing side, we closely follow Gomes, Jermann and Schmid (2016). However, our model is static. Empirical findings in Kalemli-Ozcan,

denominated in the currency of country j. We also denote by $B_t = (B_{j,t})_{j=1}^N$ the vector of debt stocks in different currencies That is, $B_{j,t}$ is the *face value of debt in currency j to be paid back at time* $t+1$. As in Gomes, Jermann and Schmid (2016), we assume that coupon payments are shielded from taxes so that

$$\mathcal{B}_{t+1}(B_t) \;=\; ((1-\tau)c+1)\sum_{j=1}^{N}\mathcal{E}_{j,t+1}B_{j,t}$$

is the total debt servicing cost, net of tax shields. Therefore, the choice of firm leverage depends on the trade-off between tax advantages and distress costs.[12] Thus, absent default, the nominal distribution to shareholders at time $t+1$ is given by

$$\Omega_{t+1}Z_{t+1} \;-\; \mathcal{B}_{t+1}(B_t) \;=\; Z_t\Omega_{t+1}Y_{t+1}\,.$$

If the idiosyncratic shock realization, $Y_{t+1} \;=\; Z_{t+1}/Z_t$, is below an endogenous default threshold $\Psi_{t+1}(B_t)$, shareholders optimally default on their debt. Upon default, shareholders get zero, debt holders takeover the firm and are able to recover a fraction $\rho < 1$ of debt face value and coupon. Thus, the dollar value of cash flows to debt-holders of currency-j debt are given by $(1+c)(\mathbf{1}_{Y_{t+1}\geq\Psi_{t+1}} \;+\; \rho\,\mathbf{1}_{Y_{t+1}<\Psi_{t+1}})\mathcal{E}_{j,t+1}$. Hence, by direct calculation, using the fact that the idiosyncratic shocks Y_{t+1} are independent of $\Omega_{t+1},\; M_{t,t+1},\; \mathcal{E}_{j,t+1}$, we get that the dollar price of one unit of debt denominated in currency j is given by

$$\begin{aligned}\delta^j(B_t) \;&=\; E_t[M_{t,t+1}(\mathbf{1}_{Y_{t+1}\geq\Psi_{t+1}} \;+\; \rho\,\mathbf{1}_{Y_{t+1}<\Psi_{t+1}})(1+c)\mathcal{E}_{j,t+1}] \\ &=\; E_t\left[M_{t,t+1}\left(1-(1-\rho)\Phi(\Psi_{t+1}(B_t))\right)(1+c)\mathcal{E}_{j,t+1}\right]\;,\end{aligned}$$

where $\Phi(\Psi_{t+1}(B_t))$ is the default probability conditional on the realization of aggregate

Liu and Shim (2018) suggest that the effects of foreign currency debt on firms' behavior might be even stronger in a dynamic setting.

[12]For simplicity, as in Gomes, Jermann and Schmid (2016), we assume that tax shields are the only motivation for issuing debt. However, one can also interpret τ as a reduced form of gains from debt issuance, such as the alleviation of adverse selection costs.

variables.[13] We assume that firms face a proportional cost $q(j)$ of issuing in country j currency for $j = 1, \cdots, N$[14] and maximize equity value plus the proceeds from the debt issuance net of issuance costs. Thus, *conditional on no default,* the equity value V_t of a given firm *after the previous period debt had been repaid* satisfies

$$V_t \;=\; \Omega_t Z_t \;+\; \max_{B_t} \left\{ \sum_{j=1}^{N} \delta^j(B_t) B_{j,t}(1-q(j)) + E_t[M_{t,t+1} \max\{V_{t+1} \;-\; \mathcal{B}_{t+1}(B_t), 0\}] \right\}.$$

It is then straightforward to show that equity value is homogeneous in Z_t, so that $V_t = Z_t \bar{\Omega}_t$ for some variable $\bar{\Omega}_t$ that is independent of idiosyncratic shocks. Thus, default occurs whenever Y_{t+1} falls below the default threshold

$$\Psi_{t+1}(B_t) \;\equiv\; \frac{\mathcal{B}_{t+1}(B_t)}{\bar{\Omega}_{t+1}},$$

Note that, importantly, Ψ_{t+1} is *invariant to currency choice* because both the numerator $\mathcal{B}_{t+1}(B_t)$ and the denominator $\bar{\Omega}_{t+1}$ are denominated in dollars.[15]

Everywhere in the sequel, we use $E_t^{\$}$ and $\mathrm{Cov}_t^{\$}$ to denote conditional expectation and covariance under the dollar risk neutral measure with the conditional density $E_t[M_{t,t+1}]^{-1} M_{t,t+1}$.

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[13] Let $X_{t+1} = (\Omega_{t+1},\ M_{t,t+1},\ \mathcal{E}_{j,t+1})$. Then, by the law of iterated expectations,

$$\begin{aligned} & E_t[M_{t,t+1}(\mathbf{1}_{Y_{t+1} \geq \Psi_{t+1}} \;+\; \rho\, \mathbf{1}_{Y_{t+1} < \Psi_{t+1}})(1+c)\mathcal{E}_{j,t+1}] \;=\; E_t[M_{t,t+1}(1 - \mathbf{1}_{Y_{t+1} \leq \Psi_{t+1}} \;+\; \rho\, \mathbf{1}_{Y_{t+1} < \Psi_{t+1}})(1+c)\mathcal{E}_{j,t+1}] \\ &=\; E_t[M_{t,t+1}(1-(1-\rho)E[\mathbf{1}_{Y_{t+1} \leq \Psi_{t+1}} | X_{t+1}])(1+c)\mathcal{E}_{j,t+1}] \;=\; E_t\left[M_{t,t+1}\left(1-(1-\rho)\Phi(\Psi_{t+1}(B_t))\right)(1+c)\mathcal{E}_{j,t+1}\right] \end{aligned}$$

[14] While we do not micro-found these costs, it is not difficult to do so. These costs may originate from underwriting costs, the limited risk bearing capacity of intermediaries (in the case of bank loans), or the actual debt placement costs incurred by the investment banks (such as locating bond investors). The observed convenience yields for dollar-denominated debt (Jiang, Krishnamurthy and Lustig (forthcoming)) can be viewed as a negative issuance cost $q(\$)$ in our model.

[15] In particular, the currency-k price of debt denominated in currency j satisfies $\delta^j(B_t, k) = E_t[M_{t,t+1}^k\left(1-(1-\rho)\Phi(\Psi_{t+1}(B_t))\right)(1+c)\mathcal{E}_{j,t+1}/\mathcal{E}_{k,t+1}]$ where $M_{t,t+1}^k = M_{t,t+1}^{\$}\mathcal{E}_{k,t,t+1}$ is the pricing kernel in currency k.

Furthermore, for each stochastic process X_t, we consistently use the notation

$$X_{t,t+1} \equiv \frac{X_{t+1}}{X_t}.$$

We need the following assumption to ensure that the leverage choice problem has a non-trivial solution.

Assumption 1 *The (exogenously specified) issuance costs $q(j),\ j = 1, \cdots, N$ satisfy*

$$\begin{aligned} (1-q(j))(1+c) &> (1+c(1-\tau)) \qquad \textit{and} \\ \bar{q}(j,\$) &\equiv \frac{((1-q(j))(1+c)-(1+c(1-\tau)))}{(1-\rho)(1+c)[(1-q(j))+\ell(1-q(\$))]-(1+c(1-\tau))} > 0 \end{aligned}$$

for all $j = 1, \cdots, N$. We also define $\bar{q}(\$) \equiv \bar{q}(\$,\$)$.

The first condition ensures that the cost $q(j)$ of issuing debt is less than the gains, as measured by the value of tax shields, so there is positive debt issuance. The second condition ensures that the recovery rate ρ is sufficiently low: Otherwise, funding becomes so cheap for the firm that the firm may want to issue infinite amounts of debt. The following is true.[16]

Theorem 1.1 *Issuing debt only in dollars is optimal if and only if*

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} - 1 \leq \frac{\mathrm{Cov}_t^{\$}\left(\bar{\Omega}_{t+1}^{-\ell}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[\bar{\Omega}_{t+1}^{-\ell}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]} \tag{1}$$

[16] As we show in the Appendix (see Proposition A.1), in our model, firms never hedge their foreign exchange risk. There is ample evidence that firms often choose not to hedge their foreign currency risk. See, for example, Bodnár (2006) who shows that only 4% of Hungarian firms with foreign currency debt hedge their currency risk exposure. Furthermore, according to Salomao and Varela (2020): "data from the Central Bank of Peru reveals that only 6% of firms borrowing in foreign currency employ financial instruments to hedge the exchange rate risk, and a similar number is found in Brazil." Du and Schreger (2017) also provide evidence that firms do not fully hedge their currency risk exposures. See also Niepmann and Schmidt-Eisenlohr (2019), Bruno and Shin (2017). That being said, Liao (2020) does find evidence that at least 40% of global firms issue currency-hedged foreign debt. While it is known that costly external financing makes hedging optimal (see, for example, Froot, Scharfstein and Stein (1993) and Hugonnier, Malamud and Morellec (2015)), Rampini, Sufi and Viswanathan (2014) show both theoretically and empirically that, in fact, more financially constrained firms hedge less. For more on hedging, see Section 2.4.

for all $j = 1, \cdots, N$ *. In this case, optimal dollar debt satisfies*

$$B_{\$,t} \;=\; (1 + c(1-\tau))^{-1} \left(\frac{\bar{q}(\$)}{E_t^{\$}\left[\bar{\Omega}_{t+1}^{-\ell}\right]} \right)^{\ell^{-1}} .$$

Absent heterogeneity in issuance costs (that is, when $q(j)$ is independent of j), (1) takes the form of

$$\mathrm{Cov}_t^{\$}\left(\bar{\Omega}_{t+1}^{-\ell}, \mathcal{E}_{j,t,t+1}\right) \;\geq\; 0 \;,\; j = 1, \cdots, N \,. \tag{2}$$

Intuitively, at time t, firms, when deciding on the currency composition of their debt, choose to issue in dollars if they anticipate the dollar to depreciate at those times when their time $t+1$ valuation is low; condition (2) provides a precise formalization of this intuition. Since $\left(\bar{\Omega}_{t+1}\right)^{-\ell}$ attains its largest value when $\bar{\Omega}_{t+1}$ is close to zero, covariance (2) overweighs the distress states: When ℓ is sufficiently high, (2) essentially requires the dollar to depreciate against all its key competitors during times of major economic downturns.

It is also important to note that condition (2) corresponds to the problem a firm faces when choosing between dollar debt and debt denominated in other key currencies, such as, e.g., the euro, the yen, the Swiss franc, and the pound. For an emerging markets' firm that is choosing between local currency debt and dollar debt, heterogeneity in issuance costs may be as (if not more so) important as the currency risk profile. However, even for the choice between dollar- and euro-denominated debt, ignoring differences in issuance costs puts the dollar at a disadvantage: Existing evidence (see, e.g., Velandia and Cabral (2017)) suggests that issuing debt in dollars is significantly cheaper than issuing in euros.[17]

To test the validity of condition 1, we need to find an empirical proxy for $\bar{\Omega}_t$. We suppose for simplicity that the distressed state only lasts for one period, and debt holders run the

[17] See also Jiang, Krishnamurthy and Lustig (forthcoming) and Liao (2020): Dollar convenience yield effectively implies negative issuance costs for dollar-denominated debt.

firm inefficiently, making its output drop. We call this drop "Distress Costs". The following is true.

Proposition 1.2 *Let S_t be the (value-weighted) stock market index (i.e., total market capitalization of all (large and diversified) firms. Then,*

$$S_t \;=\; \bar{\Omega}_t \;-\; Distress\ Costs_t\,.$$

Proposition 1.2 shows that $\bar{\Omega}_t$ is closely related to the market portfolio. If distress costs are small relative to the total value of the stock index, then $\bar{\Omega}_t$ can be directly proxied by the corresponding stock market index of "similar" firms. We will therefore use stock market index returns in our empirical tests of condition (2).

1.2 Roles for Inflation and Monetary Policy

In this section, we derive a link between the characterization in Theorem 1.1 and the inflation risk premium and assign a role for monetary policy in determining the dominant currency in debt issuance.

Denote by $\mathcal{P}_{i,t}$ inflation in country $i,\ i = 1, \cdots, N$. We will make the following assumptions about the joint long-term dynamics of inflation and exchange rates at horizons of average debt maturity.

Assumption 2 *There exists a global business cycle shock, a_t, such that*

- *Relative PPP is an important driver of exchange rates:*

$$\mathcal{E}_{j,t,t+1} \;=\; \mathcal{P}_{\$,t,t+1}\mathcal{P}^{-1}_{j,t,t+1}\, e^{\varepsilon^*_{j,t+1}}$$

where $\varepsilon^*_{j,t+1} \sim N(0, \sigma^2_{i,*})$ are the log real exchange rates

- *Stochastic discount factor is counter-cyclical,*

$$\log M_{t,t+1} \;=\; -\gamma a_{t+1} \;+\; \varepsilon^M_{t+1},$$

where $\varepsilon^M_{i,t+1} \sim N(0, \sigma^2_M)$

- *Stock prices are cyclical*

$$\log S_{i,t,t+1} \;=\; \beta_i \, a_t \;+\; \varepsilon^S_{i,t}$$

where $\varepsilon^S_{i,t} \sim N(0, \sigma^2_S)$ *and* $\beta_i > 0.$

- *all variables* $a_t,\ \varepsilon^M_{i,t},\ \varepsilon^*_{j,t},\ \varepsilon_{i,t},\ \varepsilon^S_t$ *are independent.*

Under Assumption 2, a key driver of debt currency choice will be forward-looking expectations about the co-movement of inflation with the stock market. The latter can be backed out from the inflation risk premium, given by the difference between inflation expectations under the risk-neutral and the physical measures:

$$IRP_{i,t} \;=\; \log\left(\frac{E^i_t[\mathcal{P}_{i,t,t+1}]}{E_t[\mathcal{P}_{i,t,t+1}]}\right) \;=\; \log\left(\frac{e^{r_t}\mathrm{Cov}_t(M_{i,t,t+1}, \mathcal{P}_{i,t,t+1})}{E_t[\mathcal{P}_{i,t,t+1}]}\right).$$

The following is true.

Theorem 1.3 *Under Assumption 2, firms issue all debt in US dollars if and only if US has the highest inflation risk premium.*

While Assumption 2 required to derive Theorem 1.3 is restrictive, it allows us to highlight the important link between our results and Fisherian debt deflation theory (Fisher (1933)). The common shock structure in Assumption 2 allows us to abstract from the choice between local currency and foreign currency debt, and focus on the choice between different global currencies (such as, e.g., the euro and the dollar).

2 Evidence from Forward-Looking Measures

The first goal of this section is to check whether the risk properties of the dollar fit the predictions of our theory of the dominant currency using forward-looking measures of the covariance between the stock market returns and exchange rates. The second goal is to understand the pre- and post-crisis trends in the shares of euro- and dollar-denominated debt through the lens of our model.

An ideal test of our predictions would be to test the following condition:

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} - 1 \leq \frac{\mathrm{Cov}_t^{\$}\left(\left(\bar{\Omega}_{t+1}\right)^{-\ell}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[(\bar{\Omega}_{t+1})^{-\ell}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]},$$

According to our model, abstracting from differences in issuance costs, the currency that *market participants anticipate to co-move* more with the stock market would be chosen by firms as the currency to denominate their debt.[18] We show, using forward-looking risk premia recovered from asset prices, that the dollar fits this description.

Moreover, if the distribution of issuance costs stays roughly constant across firms, our model implies a tight link between the time variation in this anticipated co-movement and the currency denomination of debt issuance. In particular, all else constant, our model would attribute the recent rise in the share of dollar-denominated debt to heightened expectations of market participants of the dollar becoming more positively (or less negatively) correlated with the stock market than the euro. This would mean that the dollar becomes more of a hedge for borrowers rather than investors. We provide evidence for the link between debt issuance patterns and such forward-looking market expectations and find support to the debt view, suggesting that firms issue more dollar debt when the dollar becomes riskier from the investors' point of view.

[18]Note that assuming that issuing dollar debt cheaper would mean that even for some negative values of the covariance, the dollar could be chosen as the currency to denominate debt. Note also that the convenience yield of dollar-denominated debt (see, Jiang, Krishnamurthy and Lustig (forthcoming)) implies that the effective issuance cost $\bar{q}(\$)$ might even be negative.

2.1 Quanto-implied risk premia and inflation risk premia

A direct way of computing the forward-looking covariance between the stock market and exchange rates is by using so-called quanto forward contracts (Kremens and Martin (2019)), which provides an almost ideal test of our theory. A euro-quanto forward contract for S&P 500 with maturity T, for example, pays off the level of the S&P 500 index *in euros*. This means that at initiation, the exchange rate is fixed. As opposed to a contract that pays off the S&P 500 in dollars, the value of this contract depends on the anticipated covariance between the index and the EUR/USD exchange rate.

Hence, the price of this contract reflects the expectations of investors about currency returns. For example, if a quanto contract on the S&P 500 denominated in euros is more valuable than the S&P 500 denominated in dollars, it means that investors expect the euro to depreciate when the index (in dollars) is low, and vice versa.

Formally, we define the quanto-implied risk premium (QRP) as:

$$QRP_t = \text{Cov}_t^{\$}\left(S_{t+1}, \frac{EUR}{USD}\right) \;=\; \frac{R_{f,t}^{\$}}{R_{f,t}^{i} P_t}(Q_t - F_t)\,, \tag{3}$$

where Q_t and F_t are quanto and vanilla forward prices, respectively.

Using the approximation

$$\text{Cov}_t^{\$}\left(S_{t+1}^{-\ell}, \frac{EUR}{USD}\right) \;\approx\; -\ell\,\text{Cov}_t^{\$}\left(S_{t+1}, \frac{EUR}{USD}\right)\,,$$

we can transform the QRP measure into the covariance we have derived in the previous section. Namely, a negative QRP means that investors expect the dollar to depreciate against the euro when the S&P 500 falls.

Kremens and Martin (2019) compute the quanto-implied covariance for contracts with a two-year maturity and find that the quanto-implied covariance of the EUR/USD exchange rate with S&P500 exhibited a very strong downward trend in the post-crisis period and has become negative in the recent years (Figure 2). This evidence is perfectly in line with the predictions of the debt view.

While quanto-implied covariance is the most relevant measure for our purposes, data obtained from Kremens and Martin (2019) only cover a period between December 2009 and October 2015. Since our goal is to explain the fall and the rise of the dollar in debt markets over the last two decades, we also resort to a longer time series containing similar information about forward-looking covariances. We use our model to generate similar predictions that we can test with other available data measuring forward-looking risk premia. A second limitation of the QRP data is that liquid quanto contracts only exist for maturities of two years and lower. To remedy that, we provide further evidence for the covariance over longer horizons using backward-looking measures in Section 3. Finally, quanto-contracts give us information about the covariance of the exchange rate and the S&P 500. Since our theory mainly applies to global firms that are exposed mostly to global shocks, we believe S&P 500 provides a proxy. In Section 3, we also provide backward-looking evidence on the covariances using the MSCI All Country World Index.

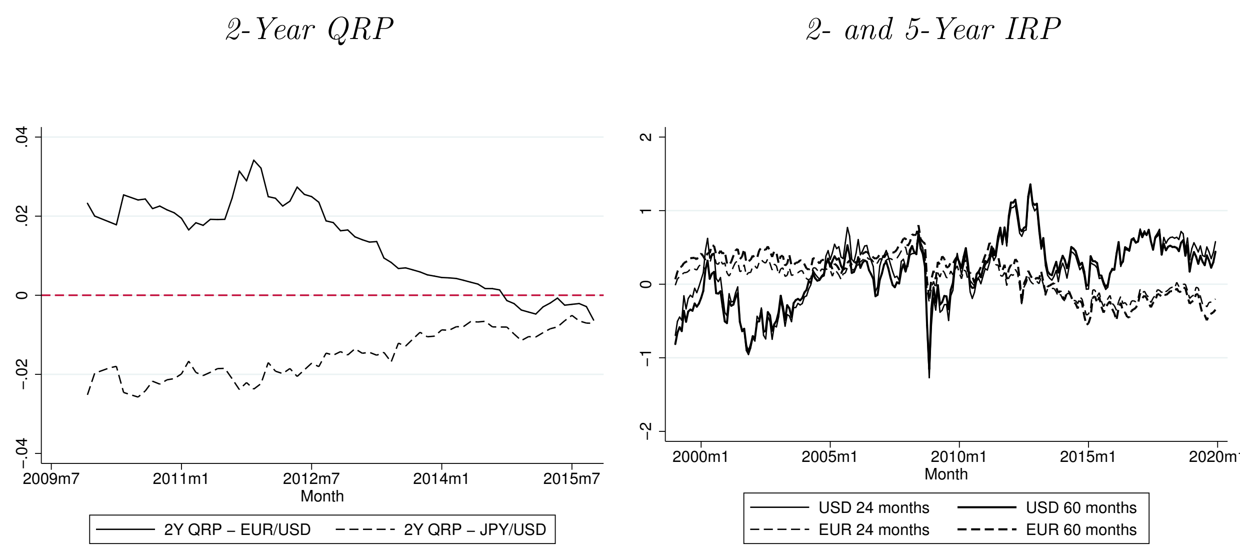
In order to obtain long time series containing information about forward-looking covariances, we appeal to Theorem 1.3 that provides a direct link between debt currency denomination and anticipated *relative* inflation cyclicality under the assumption that relative PPP is an important determinant of exchange rates at horizons of average debt maturity.[19]

The explicit link between relative inflation dynamics and exchange rates is the key element behind Theorem 1.3. Consistent with the predictions of Theorems 1.1 and 1.3, both QRP and IRP measures point to the dollar being the dominant currency in the post-crisis period. The left panel of Figure 2 shows the quanto-implied risk premium for the EUR/USD exchange rate taken directly from Kremens and Martin (2019). The right-hand panel of Figure 2

[19]While the perfect link between exchange rates and inflation relies on a strong form of PPP, Theorem 1.3 would still hold true even with large PPP deviations, as long as the relative inflation component of the exchange rates contributed significantly to the covariance (2) over the horizons of debt maturity of a typical firm. See Chernov and Creal (forthcoming) for evidence that PPP is an important driver of long-horizon currency risk premia.

shows the inflation risk premia for the euro and the dollar for two years and for five years, taken directly from Hördahl and Tristani (2014).[20] As we can see clearly from this figure, in recent years, QRP entered the negative territory whereas the dollar IRP went up relative to the euro QRP. Finally, it is important to note that Theorem 1.3 does not require US inflation to be counter-cyclical: What matters is the *relative inflation dynamics between the two countries*. Even if US inflation is procyclical, firms will still issue in dollars if they anticipate euro inflation to be lower in bad times than the US inflation.[21]

Figure 2: Two-year quanto-implied risk premium and two- and five- year inflation risk premia in the US and the Eurozone



Source: QRP data are from Kremens and Martin (2019) and IRP data are from Hördahl and Tristani (2014).

[20]The same pattern is present for longer maturities. Moreover, the difference between the dollar and the euro is more pronounced for longer maturities.

[21]In a recent paper, Campbell et al. (forthcoming) (see also Baele et al. (2007)) find that US inflation became procyclical after 2001. The fact that the dollar has a higher IRP that the euro in the post-crisis period suggests that investors anticipate US inflation to be less procyclical than that in the Eurozone. Note also Campbell et al. (forthcoming) investigate inflation cyclicality over short horizons below one year. Our estimates of the covariance of CPI changes with stock market returns suggest that this covariance is consistently negative at horizons beyond three years; that is, inflation is counter-cyclical at horizons of debt maturity.

2.2 The fall and the rise of the dollar

In this section, we state and test two predictions linking QRP and IRP dynamics with the currency denomination of debt issuance. We start with the following important result.

Proposition 2.1 *If the distribution of debt issuance costs across firms is constant over time, then the quanto-implied covariance* (3) *is* **negatively** *related to the share of dollar-denominated debt issuance.*

Proposition 2.1 is a direct consequence of Theorem 1.1: When (3) drops, the fraction of firms for which (1) holds increases and, hence, so does the dollar issuance.[22]. An analogous result holds for IRP:[23]

Proposition 2.2 *Under Assumption 2, if the distribution of debt issuance costs across firms is constant over time, then the dollar debt share is* **positively** *related to the dollar IRP and is* **negatively** *related to the euro IRP.*

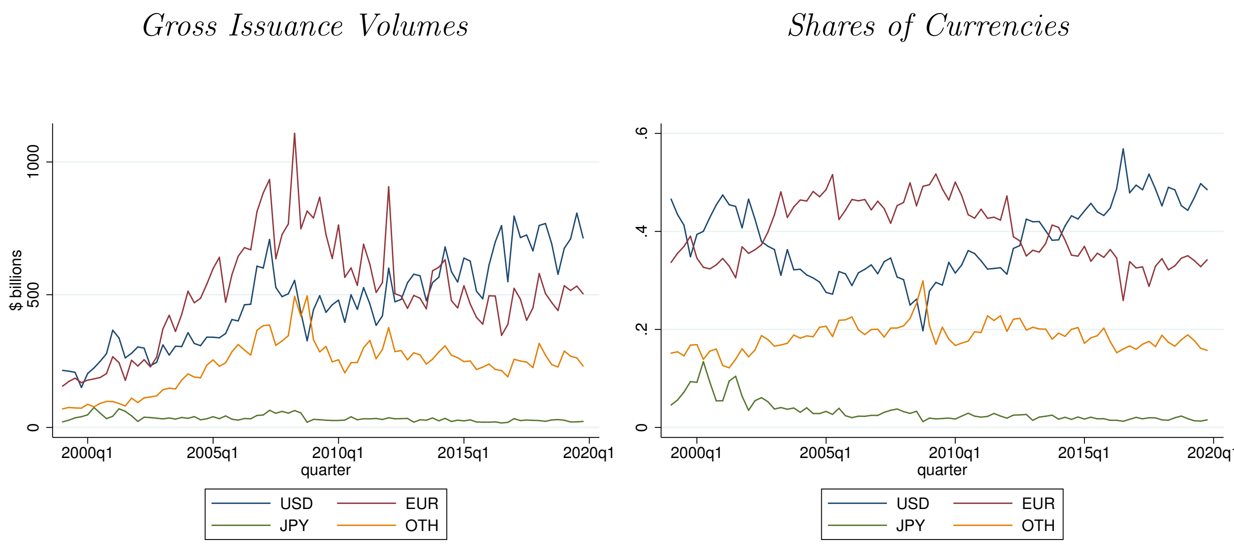
In Figure 3, we show the volume and the currency composition of gross issuance patterns of international debt as well as the shares of the dollar and the euro obtained from the BIS International Debt Securities statistics (data includes all sectors except the government).

Broad trends shown in Figure 1 for amounts outstanding, in Figure 3 for debt issuance, and in Maggiori, Neiman and Schreger (2019) for corporate debt holdings seem to be aligned with the trends in QRP and IRP documented in Figure 2. These joint dynamics of risk premia and dollar debt quantities lend support to the debt view (Predictions 2.1 and 2.2).

[22] Note that even if the quanto-implied covariance (3) is positive (as it was before 2015), firms may still prefer issuing in dollars as long as it is cheaper than issuing in euro. For example, according to Velandia and Cabral (2017), "... in the case of Mexico, the average bid-ask spread of the yield to maturity on outstanding dollar-denominated international bonds is seven basis points, compared to 10 basis points for outstanding euro-denominated bonds. Mexico is also an example with very liquid benchmarks on both currencies." The dollar convenience yield (Jiang, Krishnamurthy and Lustig (forthcoming)) further amplifies this effect.

[23] In our model, IRP can be viewed as a barometer of market expectations about inflation cyclicality. While our model is silent about the origins of these expectations, one might speculate that the observed pattern in inflation risk premia between the euro and the dollar, shown in Figure 2, may be due to declining expectations of inflation stabilization and an increasing expected risk of deflation in Eurozone following the Global Financial Crisis in 2008 and the European Debt Crisis in 2011.

Figure 3: Gross Issuance of International Debt Securities



Source: BIS International Debt Securities

In particular, in the pre-crisis period, IRP for the euro was higher than the IRP for the dollar, and after the crisis, this relationship reversed. In line with our predictions, the share of dollar debt was in decline before the crisis and increased after the crisis. Moreover, during the period for which we have data, the QRP of the euro against the dollar declined strongly.

Both the QRP and the IRP dynamics suggest that debt holders caring about inflation and foreign exchange risk should dislike holding dollar-denominated debt and prefer holding euro-denominated debt in the post-crisis period. However, the debt view implies that firms will still prefer issuing debt in dollars because of the attractive risk properties of the dollar.

2.3 QRP, IRP, and debt issuance dynamics

While broader trends in debt currency choice and currency and inflation risk premia are in line with our predictions, we would now like to formally test Propositions 2.1 and 2.2 about the response of dollar debt issuance to movements in risk premia. One distinctive feature of our theory is that changes to debt issuance currency are driven by expectations and could change quickly. We regress various measures of the dollar's share in debt markets on changes in risk premia at a quarterly frequency. First, we test Proposition 2.1: As the euro becomes less of a hedge for firms, i.e., QRP declines, do they issue more dollar debt? Second, we test Proposition 2.2: as the euro IRP becomes lower than the dollar IRP, do firms issue more dollar debt?

Table 1: QRP, IRP, debt currency choice

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	Full	Full	Full	Full	Full	Full
	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}
$QRP_{€/\$,t}^{2Y}$	-3.463***	-1.574**	-3.505***			
	(0.314)	(0.680)	(0.305)			
$IRP_{\$,t}^{2Y}$				-0.00612	-0.00870	0.0330
				(0.0197)	(0.0254)	(0.0232)
$IRP_{€,t}^{2Y}$				-0.201***	-0.194***	-0.200***
				(0.0266)	(0.0406)	(0.0251)
Trend		X			X	
Control			X			X
Period	09q4-15q3	09q4-15q3	09q4-15q3	99q1-19q4	99q1-19q4	99q1-19q4
Observations	24	24	24	84	84	84
R^2	0.713	0.783	0.781	0.340	0.341	0.449

Notes: Robust standard errors are shown in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. Debt issuance data includes all sectors except the government. Latest observed values of $QRP_{€/\$,t}^{2Y}$, $IRP_{\$,t}^{2Y}$ and $IRP_{€,t}^{2Y}$ in a given quarter are used. $QRP_{€/\$,t}^{2Y}$ data come from Kremens and Martin (2019), and $IRP_{\$,t}^{2Y}$ and $IRP_{€,t}^{2Y}$ are recovered using the methodology in Hördahl and Tristani (2014). Trend refers to a linear time trend and control refers to the inclusion of total issuance as a control variable.

We report the results in Table 1. Column (1) shows that one standard deviation decrease in $QRP^{2Y}_{€/\$,t}$, which is around 0.01, is associated with around three percentage points higher dollar share in debt issuance in a given quarter. Note that the average of the total issuance is $1,284 billion. Hence, three percentage points amount to around $38 billion in a quarter. In column (2), we rerun the regression with a linear time trend, and in column (3), we control for total issuance. The results are qualitatively similar. In columns (4), (5), and (6), we rerun the same type of regressions for the dollar share in debt issuance and inflation risk premia in the United States and the Eurozone. The results suggest that while debt issuance patterns do not move much with inflation risk premia in the United States, they mostly react to the inflation risk premia movements in the Eurozone. Thus, the decline of the euro as a preferred currency for debt issuance might be due to rising deflation risk in the Eurozone after the European sovereign debt crisis. The magnitude is also sizable as the results suggest that a one standard deviation decrease in the Eurozone IRP (which is 0.02) corresponds to around four percentage point higher dollar share in debt issuance.[24]

The above regression results provide additional support for our theory. Moreover, the effects we identify hold in a relatively high frequency, lending support to another prediction of our theory: Changes in the currency of debt issuance respond to expectations and therefore could occur relatively quickly.

2.4 Exchange rate expectations or convenience yield: What drives debt issuance dynamics?

Issuing dollar-denominated debt is cheap, not only due to more liquid markets and lower underwriting costs (Velandia and Cabral (2017)) but also due to a significant investor demand for dollar assets that creates a convenience yield for dollar-denominated safe assets (Jiang, Krishnamurthy and Lustig (forthcoming)). Liao (2020) computes a measure that

[24] In the Appendix, we report the results for non-bank and bank debt issuance separately. The results are similar for both sectors.

captures the FX-hedged corporate borrowing cost differential, *the corporate basis*, which is effectively the convenience yield of dollar-denominated corporate debt. A higher corporate basis between the dollar and the euro means that it is cheaper to borrow in dollars than in euros, making the dollar issuance cost $q(\$)$ in equation (1) effectively negative. When this basis is changing over time, a direct analog of Proposition 2.1 implies that dollar debt issuance dynamics could potentially be driven both by the corporate basis and the hedging properties of the dollar.

In this section, we rerun the regressions in Table 1, also controlling for the corporate basis to explain the dynamics of the global dollar share of debt issuance. We report the results of this regression in Table 2. As one can see, the signs on QRP and IRP and their statistical significance remain unchanged, lending further support to the debt view. Moreover, the sign of the coefficient on the corporate basis is negative (i.e., "the wrong sign"), suggesting that, in aggregate, fluctuations in the convenience yield do not drive debt issuance dynamics.

3 Evidence from Backward-Looking Measures

Our key theoretical condition posits that firms prefer to issue in dollars if they anticipate the dollar exchange rate to positively co-move with their stock market returns over their debt maturity horizons.[25] While the quanto-implied QRP is the ideal empirical measure of such forward-looking expectations, the most extended maturity of liquid quanto contracts is two years, while the average debt maturity of a firm is typically much longer. In this subsection, we investigate backward-looking measures of the dollar-stock market co-movement, arguing that rational agents might use these measures as a basis for their forward-looking expectations.

We use two stock market indices to test our predictions, namely S&P 500 and the MSCI

[25]In fact, given that issuing in dollars is cheaper than issuing in any other currency, condition (1) implies that firms would issue all their debt in dollars even if this correlation were negative, but not too negative relative to the cost gain of issuing in dollars.

Table 2: QRP, IRP, corporate basis, and debt currency choice

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	USD^{shr}_t	USD^{shr}_t	USD^{shr}_t	USD^{shr}_t	USD^{shr}_t	USD^{shr}_t	USD^{shr}_t
$QRP^{2Y}_{€/\$,t}$	-3.463***			-1.566**			
	(0.314)			(0.718)			
$Corp.Basis_t$		-0.00113	-0.00295***	-0.00182**			-0.000846*
		(0.000786)	(0.000450)	(0.000772)			(0.000451)
$IRP^{2Y}_{\$,t}$					-0.00612	0.0393	0.0435*
					(0.0197)	(0.0259)	(0.0251)
$IRP^{2Y}_{€,t}$					-0.201***	-0.192***	-0.186***
					(0.0266)	(0.0239)	(0.0232)
Period	09q4-15q3	03q4-16q2	09q4-15q3	09q4-15q3	99q1-19q4	03q4-16q2	03q4-16q2
Observations	24	51	24	24	84	51	51
R-squared	0.713	0.053	0.747	0.783	0.340	0.527	0.555

Notes: Robust standard errors are shown in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. Debt issuance data includes all sectors except the government. Latest observed values of $QRP^{2Y}_{€/\$,t}$, $IRP^{2Y}_{\$,t}$ and $IRP^{2Y}_{€,t}$ in a given quarter are used. $QRP^{2Y}_{€/\$,t}$ data come from Kremens and Martin (2019), $IRP^{2Y}_{\$,t}$ and $IRP^{2Y}_{€,t}$ are recovered using the methodology in Hördahl and Tristani (2014), and $Corp.Basis_t$ data come from Liao (2020). Different sample periods are due to differences in data availability and multiple columns provide robustness checks by aligning sample periods.

AC World Index measured in dollars, to be consistent with our theoretical conditions. For this sub-section, we use the trade-weighted dollar index against major currencies, including those in Eurozone, Canada, Japan, United Kingdom, Switzerland, Australia, and Sweden, as obtained from the FRED database.[26] We conduct our analysis by first looking at simple covariances, then through a VAR analysis in order to gauge the lead-lag relationships between

[26]Our results are robust when we use other indices such as the narrow or the broad dollar index obtained from the BIS.

stock indices and the dollar. In the internet appendix, we also provide results using the bilateral exchange rates between the dollar and the euro,[27] the yen, the pound, and the Swiss franc.

3.1 Why is the dollar the dominant currency? Results with the dollar index

Given that the dollar is the most common currency of denomination in international debt markets, the first prediction of our model is that the returns on the dollar index positively correlate with the returns on the stock market indices at horizons that correspond to the typical debt maturity (see section 4, Choi, Hackbarth and Zechner (2018), Cortina, Didier and Schmukler (2018)). To test this prediction, we first run the following regressions for the horizons of $h \in \{3, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120\}$ months:[28]

$$Ret\_USD_{t-h,t} = \alpha^h + \beta_h Ret\_StockIndex_{t-h,t} + \epsilon_{t-h,t}\,. \tag{4}$$

Here, $Ret\_USD_{t-h,t}$ and $Ret\_StockIndex_{t-h,t}$ denote the rolling (overlapping) returns on the dollar index and the two indices we use (in two separate regressions) over h months, respectively. Figure 4 reports the results for the regression coefficient β_h for different horizons and for different stock market indices, together with the 95% confidence intervals. The sample period for the MSCI series starts in January 1988. The round dots and the corresponding solid lines in both panels represent the point estimates and 95% confidence intervals obtained using a sample period between January 1988 and December 2019. The squared dot and the corresponding dashed lines on the left-hand panel represent the point

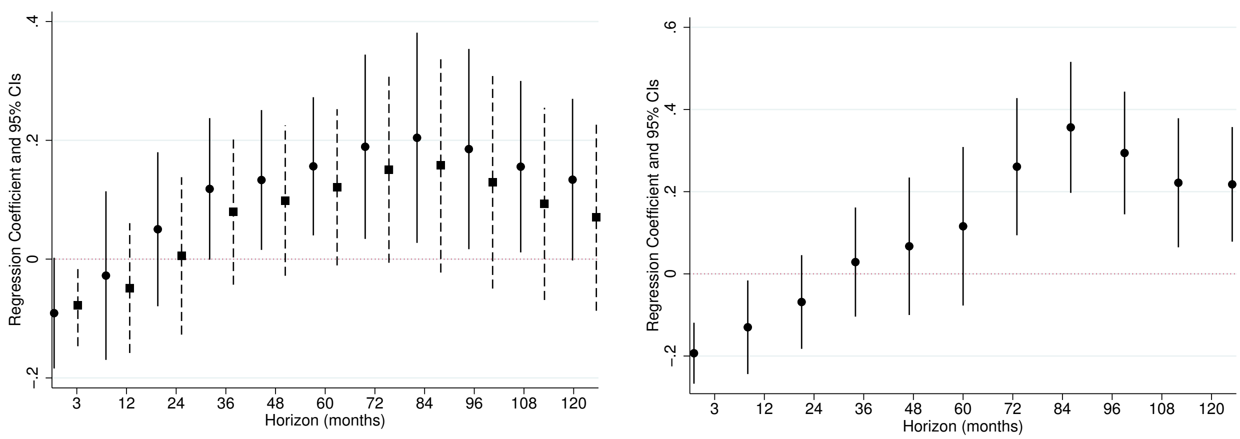
[27] We use the Deutsche mark prior to the introduction of the euro using the euro/Deutsche mark exchange rate at the time of the inception of the euro.

[28] We then control for autocorrelation at the respective horizons by using the Newey-West correction with the respective number of lags.

estimates and 95% confidence intervals obtained using a sample period between January 1973 and December 2019 for the S&P 500 and the FRED dollar index.

Figure 4: The betas of the USD index returns with respect to stock market returns

S&P 500 Index *MSCI AC World Index*



Notes: The graphs report the regression coefficients β_h from the regressions (4). Standard errors are corrected using the Newey-West procedure with the number of lags being equal to the horizon h of returns for each respective regression. The round dots and the corresponding solid lines in both panels represent the point estimates, β_h, and 95% confidence intervals obtained using a sample period between January 1988 and December 2019. The squared dot and the corresponding dashed lines on the left-hand panel represent the point estimates, β_h, and 95% confidence intervals obtained using a sample period between January 1973 and December 2019 for the S&P 500 and the FRED dollar index.

The results show a pattern of negative betas at short horizons and positive and mostly increasing betas at longer horizons. The negative betas for shorter horizons are consistent with the findings in Gourinchas, Govillot and Rey (2017) and Gourinchas (2019), who show that the dollar tends to appreciate in bad times.[29] However, Figure 4 suggests that the sign

[29] In this paper, we focus on the choices of firms and hence the medium run risk properties of the dollar, and abstract from frictions that households face. However, one can argue that in a more realistic model with more frictions and differences in relevant horizons between households and firms, the short-run appreciation in bad times provides insurance to investors with shorter horizons and safety demand, and the medium-run depreciation of the dollar provides insurance to firms with longer maturity nominal debt, thereby reinforcing the dominant international role of the dollar.

of the relationship reverts for typical horizons of debt maturity.[30] These findings, together with condition (2), suggest that global firms, whose stock returns co-move with the S&P 500 or the MSCI AC World Index, are better off if they borrow in dollars rather than in other major international currencies if their debt maturity is sufficiently long.

In the internet appendix C.4, we repeat the same exercise with non-overlapping observations. While the statistical significance is hard to establish in that case due to a remarkable drop in sample sizes, the pattern of short-term negative dollar betas, followed by positive betas at longer horizons, is present in that exercise as well.

Why does the sign of the co-movement between the dollar index and the stock market change for longer horizons? To answer this question, we decompose the covariance between the dollar and the stock market based on the additivity of log-returns: $Ret_{t-h-j,t} = R_{t-h-j,t-h} + R_{t-h,t}$ for any pair $h,\ j > 0$. Using this decomposition, we get that

$$
\begin{aligned}
&\text{Cov}(Ret\_USD_{t-h-j,t},\ Ret\_SP500_{t-h-j,t}) \\
&= \underbrace{\text{Cov}(Ret\_USD_{t-h-j,t-h},\ Ret\_SP500_{t-h-j,t-h}) + \text{Cov}(Ret\_USD_{t-h,t},\ Ret\_SP500_{t-h,t})}_{co-movement} \\
&+ \underbrace{\text{Cov}(Ret\_USD_{t-h-j,t-h},\ Ret\_SP500_{t-h-j,t})}_{USD\ leading\ S\&P500} + \underbrace{\text{Cov}(Ret\_SP500_{t-h-j,t-h},\ Ret\_USD_{t-h,t})}_{S\&P500\ leading\ USD}.
\end{aligned}
\tag{5}
$$

Since the co-movement terms in the covariance decomposition are negative for shorter horizons, while the total covariance is positive for longer horizons (see Figure 4), it has to be that at least one of the lead-lag terms in (5) is positive and sufficiently large to offset the negative co-movement terms.

[30]Interestingly enough, the same pattern of sign reversal at longer horizons is also observed in the behavior of UIP deviations. See Valchev (2015) and Engel (2016). Understanding the links between these findings and our results is an interesting direction for future research.

3.2 VAR Analysis

In order to understand the impact of the lead-lag relationships in (5) on the joint dynamics of the dollar and S&P 500, we estimate the following two-lag VAR model at annual frequency:[31]

$$\begin{pmatrix} Ret\_USD_{t-1,t} \\ Ret\_SP500_{t-1,t} \end{pmatrix} = \Psi \begin{pmatrix} Ret\_USD_{t-2,t-1} \\ Ret\_SP500_{t-2,t-1} \end{pmatrix} + \Gamma \begin{pmatrix} Ret\_USD_{t-3,t-2} \\ Ret\_SP500_{t-3,t-2} \end{pmatrix} + \varepsilon_t$$

where $\Psi = \begin{pmatrix} \Psi_{1,1} & \Psi_{1,2} \\ \Psi_{2,1} & \Psi_{2,2} \end{pmatrix}$ and $\Gamma = \begin{pmatrix} \Gamma_{1,1} & \Gamma_{1,2} \\ \Gamma_{2,1} & \Gamma_{2,2} \end{pmatrix}$ and $\varepsilon_t \sim N(0, \Sigma)$ for some 2×2 covariance matrix Σ.

The results of this VAR are reported in Table 3. We find that $\hat{\Psi}_{1,2} > 0 > \hat{\Sigma}_{1,2}$, so that the dollar and S&P 500 co-move negatively contemporaneously, but the lagged return on the S&P 500 positively predicts the dollar.[32] This finding is consistent with the decomposition (5) and Figure 4: It is this this lead-lag relationship that is responsible for the sign change in Figure 4 at longer horizons.

Next, we use our estimates from the VAR model to compute the cumulative impulse response of both the dollar and the S&P 500 following a negative shock to the S&P 500. As one can see from Figure 5, the cumulative impact of a negative shock to the S&P 500 on the dollar is a significant depreciation at a 5% level two and six years following the shock and a significant depreciation at a 10% level for most horizons.[33] In the internet appendix, we report the VAR regressions results for the MSCI World Index (for which we have data from 1988 onwards). The results are qualitatively similar, but stronger and statistically significant

[31]The two-lag VAR has been selected based on the standard Akaike information criterion (AIC).

[32]While the results are statistically significant at the 10% level for the entire sample 1973-2019, our early sample was characterized by the end of the Bretton Woods era and the associated outlier of event of extreme dollar depreciation. If we restrict our sample to 1976-2019, the coefficient is 0.15, significant at 5% level. In the paper, we report the results using the entire sample of available data, however results based on different sample periods are available upon request.

[33]Similar to the VAR coefficients, the impulse responses are also stronger if we restrict the period to 1976-2019, with statistical significance at 5% for almost all horizons. These results are also available upon request.

Table 3: A VAR(2) model of the S&P 500 and the FRED dollar index

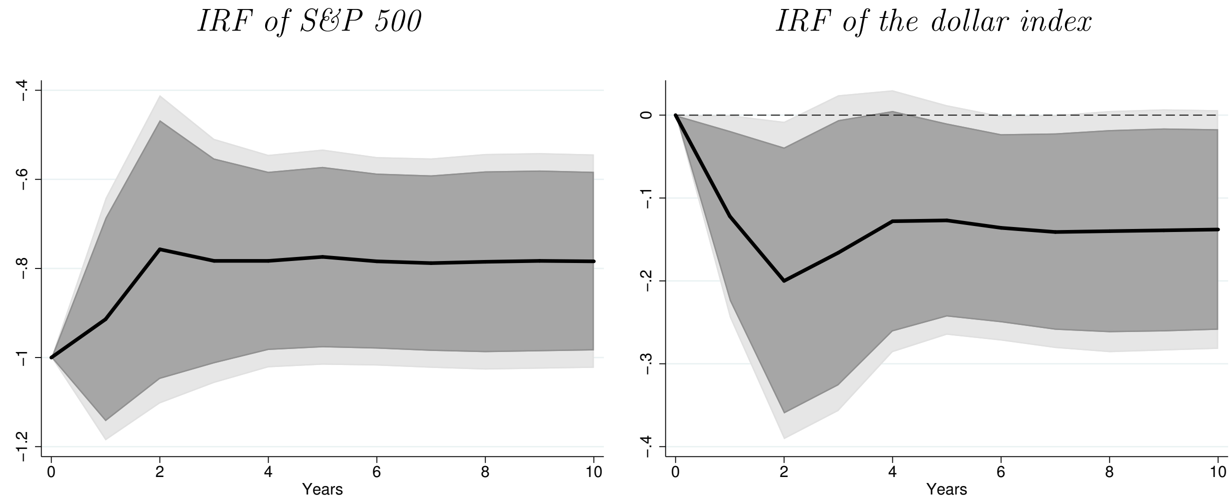
	(1)	(2)
	$Ret\_SP500_{t-1,t}$	$Ret\_USD_{t-1,t}$
$Ret\_SP500_{t-2,t-1}$	-0.0856	0.122*
	(0.147)	(0.0661)
$Ret\_SP500_{t-3,t-2}$	-0.202	0.0494
	(0.146)	(0.0657)
$Ret\_USD_{t-2,t-1}$	0.310	0.322**
	(0.335)	(0.151)
$Ret\_USD_{t-3,t-2}$	-0.240	-0.291**
	(0.329)	(0.148)
Observations	45	45
R-squared	0.0714	0.2073

Notes: Standard errors that are adjusted for small-sample degrees of freedom in parantheses. *, **, *** denote significance at the 10, 5, and 1% levels, respectively. The coefficients are from a VAR(2) model of non-overlapping annual returns on the S&P 500 and the FRED dollar index against major currencies (DTWEXM) between 1973 and 2019. The variance-covariance matrix for the error terms is estimated as: $\hat{\Sigma} = \begin{pmatrix} 0.0256 & \\ -0.0016 & 0.00516 \end{pmatrix}$

at 5% level for the MSCI World Index - in part also reflecting the stronger relationship in the later sample periods reported in Figure 4.

Figure 5 complements Figure 4, providing additional evidence for longer-run co-movement between the dollar and stock market. While historical covariance estimates in Figure 4 are based on overlapping returns, our VAR model is estimated with non-overlapping returns,

Figure 5: Cumulative Impulse Response Functions of a Shock to S&P 500



Source: Datastream, FRED, authors' calculations.
Notes: Figures show the cumulative impulse response functions of a negative 1 ppt shock to the S&P 500 based on the estimates of a VAR(2) model of the yearly returns on the S&P 500 and the FRED dollar index against major currencies (DTWEXM) between 1973 and 2019, reported in Table 3. The lines in each graph represent the cumulative impulse response functions. The darker shaded areas represent the 90% confidence intervals, while the lighter shaded areas represent the 95% confidence intervals.

allowing us to leverage a longer sample. All in all, despite significant small sample issues, our results based on historical data point towards the same direction: The choice of the dollar over other major currencies by debt issuers is consistent with the empirical evidence on the joint dynamics of the dollar and the stock market.[34]

4 Debt currency and maturity choice

Our results in Section 3 have direct implications on the link between debt maturity and the incentives to issue dollar-denominated debt. Namely, as the dollar's co-movement with the stock market increases over longer horizons, we expect that firms would not be indifferent

[34]In the internet appendix to this section, we also present results using bilateral exchange rates instead of the dollar index - results are qualitatively similar. Moreover, we discuss the international role of the pound versus the yen in relation to our theory and as supporting evidence to our mechanism.

between issuing short-term dollar debt and rolling it over and issuing long-term debt. In particular, we predict that firms would prefer issuing their longer-maturity debt in dollars.

We use data at the bond issuance level to formally test the hypothesis that the propensity to issue dollar-denominated debt increases with debt maturity. We restrict our attention to banks and non-banks separately and have a sample period between 1999 (the introduction of the euro) and the end of 2019.[35,36]

We use data from Dealogic where observations are at the ISIN level of bond issuance. To keep the timing of our analysis similar to the previous sections, we restrict the sample to bonds issued between January 1999 and December 2019. Our dataset includes a total of 706,924 bonds, issued by 61,910 firms that are headquartered in 121 different countries.

The dataset includes information on the identity of the firm, the country where it is headquartered, the industry as well as information on the bonds, such as the currency denomination, date of issuance, maturity date, issued amount denominated in the local currency of the firm's headquarters, and whether the bond is investment-grade or is not. In the full sample, the mean of the winsorized maturity is 3,408 days, with a standard deviation of 3,485 days; the minimum value is 365 days, and the maximum value is 11,681 days.

The dollar co-movement with the stock market increases over longer horizons as documented in Section 3. An implication of this result through the lens of our model is that the propensity to issue dollar-denominated debt should increase with debt maturity. Following our results in Section 3, we test the hypothesis that longer debt maturity is associated with a higher propensity to issue dollar-denominated debt using micro-level data on bond issuance.

To measure the propensity to issue dollar-denominated debt, we use $\mathbb{1}(USD)$, which is a dummy variable that takes the value one if the currency denomination of the bond is the dollar. Then, the independent variables of interest in our regressions become $Maturity_w$,

[35]The dataset includes perpetual bonds as well. In order to have them in the analysis, we winsorize the maturity of the bonds at 5%, both at the lower and upper tail of their maturity distribution. Winsorizing the maturity at 10%, 2.5% or winsorizing only the right tail of the distribution do not change the results materially.

[36]We exclude data on the government sector and focus only on private sector bond issuance.

which is the winsorized and standardized value of maturity at the 5% level. According to our hypothesis, we expect a positive coefficient for this variable.

Other control variables are the size of the issuance and a dummy variable that is equal to 1 if the bond is investment-grade. Moreover, depending on the specification, we include $Industry * Month$, $Country * Month$, and $firm * Month$ fixed effects. We cluster the standard errors at the $country * Year$ level.[37]

We run different linear regressions, varying the fixed effects used, and making different cuts of the sample in order to test the predictions of our theory. Table 4 presents the results.

The first two columns control for bond characteristics as well as $Industry * Month$ and $Country * Month$ fixed effects for non-banks (in the first column) and for banks (in the second column). The coefficient on $Maturity_w$ suggests that a one standard deviation increase in maturity increases the likelihood of the currency denomination of the bond to be dollars by 1.5 percentage points for non-banks and 3 percentage points for banks.

Next, as part of our identification strategy, we rely on firms that issue multiple bonds in at least two different currencies in a given month. This choice allows us to tightly identify that the same firm that has access to multiple markets chooses to issue the longer maturity bond in dollars as opposed to issuing in other currencies. In columns (4) and (5), we run a similar regression for non-banks and banks, respectively, with $Firm * Month$ fixed effects. While the result for non-banks is of a similar magnitude for non-banks, it is statistically insignificant. On the other hand, the result goes through for banks.

Finally, in columns (5) and (6), we further restrict the sample to firms that are from the United States, the Eurozone, Japan, Switzerland, or Great Britain and repeat the exercise in (3) and (4). This aims to address a potential concern that our results in (3) and (4) are driven by the fact that firms in emerging markets could only access dollar bond issuance markets. Focusing only on the five countries with liquid and deep capital markets alleviates

[37] In the benchmark specification, we use the country where the headquarters of the parent company of the issuer is located. As a robustness check, we use the residence of the issuer instead. Our results then are virtually unchanged.

Table 4: Debt maturity and currency choice

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	NB	B	NB	B	NB†	B†
	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$
$Maturity_w$	0.0149***	0.0299***	0.0163	0.0770***	0.0350**	0.0734***
	(0.00339)	(0.00787)	(0.0138)	(0.0126)	(0.0157)	(0.0155)
Controls	X	X	X	X	X	X
Industry*Month FE	X	X				
Country*Month FE	X	X				
Firm*Month FE			X	X	X	X
Observations	457,266	243,657	46,562	67,213	31,857	49,268
R-squared	0.684	0.665	0.450	0.432	0.475	0.472
Mean of Dep. Var	0.556	0.534	0.376	0.348	0.430	0.350

Notes: Standard errors clustered by $Country * Year$ in parantheses. *, **, *** denote significance at the 10, 5, and 1% levels, respectively. $\mathbb{1}(USD)$ is a dummy variable that takes the value 1 if the currency of the issued bond is the dollar. $Maturity_w$ is the standardized value of maturity winsorized at 5% and 95% levels. Controls include the size of the issuance and a dummy variable for the status of investment-grade status of the bond. NB refers to the sample of non-bank financials and non-financial corporations. B refers to the sample of banks. Columns (3), (4), (5), and (6) only include firms that issue in at least two currencies in a given month. † means that the sample is further restricted only to those firms that are from the United States, the Eurozone, Japan, Switzerland or the Great Britain.

this concern as these firms could potentially issue in their home currency or any other major currency. The results from these regressions are in line with our hypotheses both for banks and non-banks.

5 Dynamic Capital Structure Choice

Our main theoretical result in Theorem 1.1 is based on the assumption that capital structure choice is static. That is, when a firm issues debt at time $t = 0$, it does not reoptimize until the debt expires. In reality, it is possible that a firm dynamically readjusts its capital structure by frequently buying back old debt or issuing new debt. If that is the case, a key concern could be that the maturity of originally issued debt may not be the relevant horizon to evaluate the correlation of cash flows and debt value. This concern would be all the more important due to the difference of the sign of the covariance between the dollar and the stock market at short versus long horizons.[38]

In this section, we investigate the consequences of allowing firms to adjust their capital structure dynamically. Accounting for this case is a theoretical challenge due to complexities that arise from non-linear feedback effects from policy functions (i.e., leverage choice) in the future into capital structure choice in the current period. We present the first theoretical characterization of dynamic capital structure decisions with the currency composition of debt to the best of our knowledge. We derive sufficient conditions that lead to the dollar's dominance in a dynamic setup and provide further empirical evidence in line with the theory.

5.1 Theory

We extend our baseline model by introducing an intermediate period at which firms can rebalance their debt by either issuing new debt or buying back old debt at its market value. Essentially, we consider a three-period model with $t = 0, 1, 2$, where the firm issues debt at

[38]While these concerns are valid, it is important to note that, as Korteweg, Schwert and Strebulaev (2018) show, the average firm rebalances its capital structure once every five quarters. According to the results in Section 3, the safe haven properties of the dollar do not last for five quarters, partially alleviating these concerns. Moreover, Korteweg, Schwert and Strebulaev (2018) show that about 50% of firms issue long-term debt and rebalance their capital structure infrequently, while 17% of firms never adjust leverage. These facts suggest that the static capital structure choice assumption is roughly consistent with many firms' decisions in the data.

$t = 0$ that matures at $t = 2$. The firm generates after-tax profits Y_t at $t = 1, 2$. At $t = 1$, the firm can reoptimize its leverage decisions by calling back debt or issuing more debt.

Our first result is the following:

Proposition 5.1 (The leverage ratchet effect) *It is never optimal to buy back debt.*

This proposition is an extension of the classic leverage ratchet effect of Admati, Demarzo, Hellwig and Pfleiderer (2018) to the case of debt issued in multiple currencies. The intuition is the same as in Admati, Demarzo, Hellwig and Pfleiderer (2018): Buying back debt makes shareholder forego all the tax benefits for which the debt has been issued in the first place, making it suboptimal to reduce leverage. Thus, the optimal amount of debt is monotonically increasing (or stays constant) over time.

To characterize the optimal dynamic leverage policy, we will need several technical conditions. In what follows, we make the following assumptions:

Assumption 3 *We have*

- $\tau - q(j)$ *is small for all* j
- *cash flows* Y_1 *are small relative to* Y_2
- $Y_2 = \Omega_2 Z_1 Z_2$ *where idiosyncratic shocks* Z_t *are i.i.d. with the density* $\ell Z^{\ell-1}$ *on* $[0, 1]$, *and are independent of aggregate shocks.*
- *For any aggregate shock realization at time* $t = 1$, *the best firm (i.e., the firm with* $Z_1 = 1$*) does not default (that is, some firms survive even the worst crisis).*
- *The "marginal" firm with a* Z_1 *realization that is indifferent between defaulting and not defaulting at* $t = 1$ *does not issue new debt at time* $t = 1$.[39]

[39] The last two conditions are natural and can be formulated as parametric restrictions on the underlying aggregate shocks. See Appendix D.

We will denote by $r_t(j) = -\log E[M_t \mathcal{E}_{j,t}]$ the country-j nominal interest rate for horizon t. The following theorem derives conditions for the optimality of issuing dollar debt both at $t = 0$ and $t = 1$:

Theorem 5.2 *Issuing only dollar debt both at time $t = 0$ and $t = 1$ is optimal if and only if the following two conditions holds:*

- $\mathrm{Cov}_1(\Omega_2^{-\ell}, \mathcal{E}_{j,2}) \geq 0$ *for all j.*
- *we have*

$$\frac{q(\$) - q(j)}{\tau - q(\$)} \;\leq\; \frac{\mathrm{Cov}^{\$}(\Omega_2^{-\ell}, \mathcal{E}_{j,2})}{E^{\$}[\Omega_2^{-\ell}]E^{\$}[\mathcal{E}_{j,2}]} \;+\; \frac{E^{\$}[\Omega_2^{-\ell}\mathcal{E}_{j,2}]}{E^{\$}[\Omega_2^{-\ell}]E^{\$}[\mathcal{E}_{j,2}]} \frac{c_1(e^{r_2(\$)-r_1(\$)} - e^{r_2(j)-r_1(j)})}{c_1 e^{r_2(j)-r_1(j)} + c_2}$$

In particular, if $q(\$) = q(j)$ for all j, then a sufficient condition for issuing only dollar-denominated debt is that

(a) dollar is the riskiest currency so that $\mathrm{Cov}_t^{\$}(\Omega_2^{-\ell}, \mathcal{E}_{j,2}) \geq 0$ for all j and $t = 0,\ 1$ and

(b) dollar has the largest term premium: $r_2(\$) - r_1(\$) > r_2(j) - r_1(j)$.

The intuition behind Theorem 5.2 is as follows. Firms issue debt due to its tax benefits, and optimal capital structure is determined by the trade-off between the tax benefits and the risk of default. At the time $t = 1$, debt is issued to be held until it expires. Hence, Theorem 1.1 applies and the inequality $\mathrm{Cov}_1^{\$}(\Omega_2^{-\ell}, \mathcal{E}_{j,2}) \geq 0$ is necessary and sufficient for the optimality of dollar debt. The situation at $t = 0$ is more complex. From the time $t = 0$ point of view, both the tax benefits and the effective costs of default (due to the lost cash flows) accrue at times $t = 1$ and $t = 2$ when the coupon is paid. Since, by assumption, most of the cash flows occur at time $t = 2$, this is also where the highest costs of default occur. Thus, the trade-off can be decomposed as:

$$trade-off \;\approx\; tax\ benefits(t=1) \;+\; tax\ benefits(t=2) \;-\; default\ cost(t=2)\,.$$

The default cost itself can be decomposed further into

$$default\,cost(t=2) \;=\; average\,cost(t=2) + cost\ from\ co\!-\!movement\ with\ FX\,(t=2)\,.$$

As in Theorem 1.1, the trade-off between $tax\ benefits(t=2)$ and the $cost\ from\ co-movement\ with\ FX(t=2)$ is determined by $\mathrm{Cov}^{\$}(\Omega_2^{-\ell}, \mathcal{E}_{j,2})$. Thus, we are left with the second, purely inter-temporal part of the trade-off:

$$intertemporal\ trade-off \;=\; tax\ benefits(t=1) - average\ cost(t=2)\,.$$

Naturally, this trade-off is determined by the slope of the yield curve: The tax benefits at time $t=1$ are discounted at the rate $r_1(j)$, while the default cost paid at time $t=2$ is discounted at the rate $r_2(j)$. The larger the difference $r_2(j) - r_1(j)$, the stronger is the incentive to issue debt in currency j, get the tax benefits at $t=1$ and pay the cost at time $t=2$.

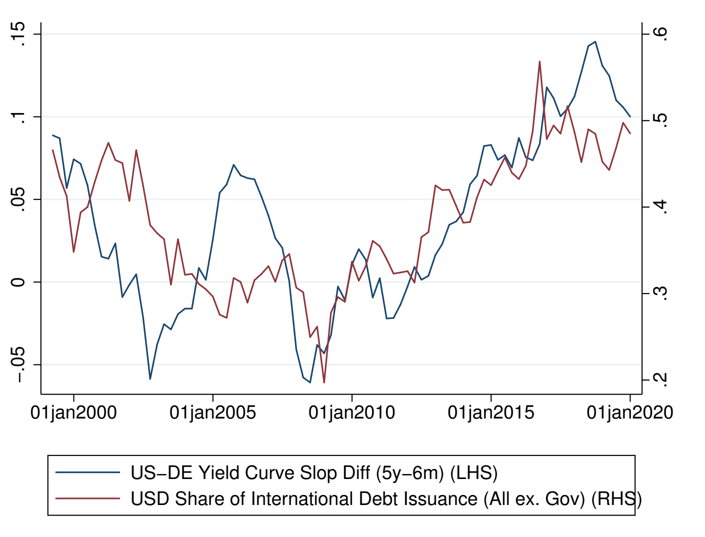
5.2 Empirical Evidence

In this section, we provide empirical evidence showing that the yield curve slope differential between the dollar and the euro is a relevant variable for the choice of currency in debt issuance[40]. We view this evidence as additional support of the "debt view": The fact that dollar debt issuance is (at least partially) driven by firms' optimal capital structure choices.

Similarly to Propositions 2.1 and 2.2, Theorem 5.2 implies a positive relationship between the yield curve slope differential, $(r_2(\$) - r_1(\$)) - (r_2(j) - r_1(j))$ and dollar debt issuance. Figure 6 illustrates that the 5y-6m difference between the US Treasury and the German bund

[40]Note that, in theory, we have $r_t(j) = -\log E[M_t \mathcal{E}_{j,t}]$ the country-j nominal interest rate for horizon t. $r_t(j)$ are the non-annualized interest rates. Therefore we scale back the yields that we obtain from the yield curves to reflect the non-annualized interest rates.

Figure 6: Yield curve slope differentials and dollar debt share



curve has largely been positive and increasing since 2008, while it was negative and at times decreasing before 2008. This is in line with the decline of the share of dollar debt before 2008 and the subsequent rise of this share after 2008 (see Figure 4).[41] Moreover, we show in a regression that these differences are positively associated with the share of dollar debt for various horizons. In Table 5, we show the results of a simple regression of the share of dollar debt issuance at a given time t on the difference between the slopes of the yield curves between the United States and Germany. Our independent variable, denoted as, $Diff^{US,DE}_{r2,r1,t}$ equals $(r_2(US) - r_1(US)) - (r_2(DE) - r_1(DE))$ in the notation of Theorem Theorem 5.2. As one can see, consistent with Theorem 5.2, the relationship is positive and highly significant for all horizon combinations.

[41] While we illustrate this only for 5y-6m, this is qualitatively similar for other horizons, such as 1y-6m, 2y-6m, 10y-6m, 2y-1y, 5y-1y, and 10y-1y.

Table 5: US-Germany yield curve slope differential and the share of dollar debt

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Dep. Var:	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}
Indep. Var:	$Diff_{12,6,t}^{US,DE}$	$Diff_{24,6,t}^{US,DE}$	$Diff_{60,6,t}^{US,DE}$	$Diff_{120,6,t}^{US,DE}$	$Diff_{24,12,t}^{US,DE}$	$Diff_{60,12,t}^{US,DE}$	$Diff_{120,12,t}^{US,DE}$
	2.539***	1.016***	0.515***	0.279***	1.633***	0.612***	0.303***
	(0.873)	(0.297)	(0.119)	(0.0637)	(0.447)	(0.134)	(0.0673)
Controls	X	X	X	X	X	X	X
Observations	84	84	84	84	84	84	84
R-squared	0.618	0.629	0.656	0.664	0.635	0.661	0.666

Notes: Robust standard errors are in parantheses. *, **, *** denote significance at the 10, 5, and 1% levels, respectively. Debt issuance data includes all sectors except the government. $Diff_{r2,r1,t}^{US,DE}$ measures the difference between the United States and Germany of the difference between the non-annualized interest rate over r_2 months and the non-annualized interest rate over r_1 months: $(r_2(US) - r_1(US)) - (r_2(DE) - r_1(DE))$. The sample period is 1999Q1-2019Q4. Each column shows the coefficient on $Diff_{r2,r1,t}^{US,DE}$ of the regression of USD_t^{shr} on $Diff_{r2,r1,t}^{US,DE}$, for different values of $r1, r2$ and controls. Controls include a time trend and total international debt issuance excluding issuance by governments in a given quarter.

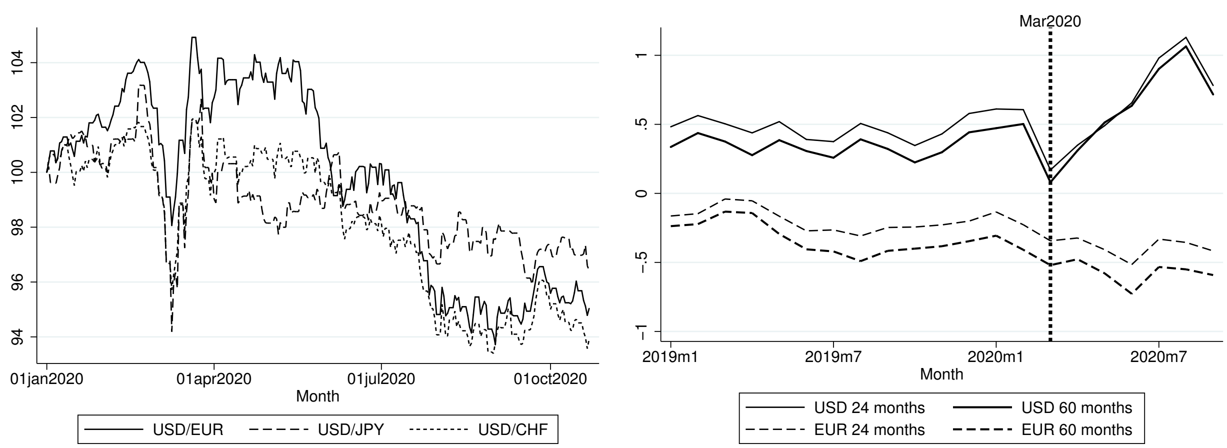
6 The Debt View during the Covid-19 Crisis and Its Aftermath

The Covid-19 crisis in March 2020 and its aftermath also provide a useful test for our theory. According to our theory, firms borrow in dollars as opposed to other major currencies due to the expectation that the dollar would depreciate following a bad global shock compared to other currencies (see Section 3.2 and Figure 5). Furthermore, the rising share of the dollar debt issuance in the post-GFC periods suggests a strengthening of this expectation from our theory's point of view.

The left-hand side of Figure 7 shows the evolution of the dollar exchange rate against the euro, Japanese yen, and the Swiss franc. Unlike previous episodes of global shocks, the dollar depreciated against these currencies without a meaningful initial appreciation. Moreover, this depreciation has been material to the order of around 10% against the euro. This highlights that the dollar's depreciation provided a hedge for firms during the Covid-19 downturn, with dollar debt providing support for our theory.

Figure 7: The dollar and inflation risk premia during the Covid-19 crisis

The dollar against major currencies *Inflation Risk Premia*



Sources: Datastream, Hördahl and Tristani (2014), authors' calculations.

The channel that runs in our theory to explain the dollar's dominance emphasizes the role played by the response of the monetary policy to shocks. During the Covid-19 crisis, the Federal Reserve reacted swiftly and forcefully to respond to the shock by re-activating a number of crisis response tools used in the GFC as well as introducing new ones, all contributing to the decline of the dollar against major currencies, reinforcing investors expectations about the ability of the US monetary policy of alleviating the debt burden of firms in bad times.

Our paper's findings might also shed light on the future of the dollar's dominance in denominating debt. Our theory suggests that the continuing divergence between the dollar and euro inflation risk premia during the Covid-19 crisis (see the right-hand panel of Figure 7) will further strengthen the dollar's international role in denominating debt contracts, consistent with Proposition 2.2.

We provide preliminary evidence that firms are more likely to issue dollar debt after the Covid-19 crisis using granular bond issuance data. We control for several factors to account for whether the same firms' propensity to issue dollar debt is higher after the Covid-19 crisis compared with other major currencies. We restrict the sample to bond issuance in five currencies: the dollar, the euro, the yen, the Swiss franc, and the pound, only in 2019 and 2020 (data retrieved from Dealogic in October 2020). We also divide the sample as pre- and post-Covid from 1 April 2020. That leaves us with 5,099 observations for banks, 4,268 of which is pre-Covid and 831 is post-Covid; 25,042 observations for non-bank financials, 18,786 of which is pre-Covid and 6,256 of which is post-Covid; and 6,640 observations for non-financial corporations, 4,136 of which is pre-Covid and 2,508 of which is post-Covid.[42]

We run a regression of whether the bond is issued in dollars or other four currencies ($\mathbb{1}(USD)$), on a dummy, $\mathbb{1}(Post\_Covid)$, which indicates whether the issue date is after 1 April 2020 as well as other control variables, such as the size of issuance, maturity, whether it is investment grade or not, and finally firm fixed effects. We present the results in Table 6. The three columns show the results for banks, non-bank financials, and non-financial corporations, respectively. The fourth column only focuses on the dollar and euro issuance by firms outside the United States and the Euro area for the full sample of banks, non-bank financials and non-financial corporations.[43] All columns suggest that all else constant, firms tended to issue more dollar debt than other major currency debt since April 2020.

[42]Note that in the regressions below, we lose some of these observations due to them being singletons in fixed effect regressions.

[43]Dividing the fourth column into different samples of the three sectors yield qualitatively similar results, but not statistically significant for non-bank financials and non-financial corporations due to large standard errors in part because of relatively small sample sizes.

Table 6: Debt issuance and currency choice: Pre- and post-Covid period

	(1)	(2)	(3)	(4)
Sample:	B	NBFI	NFC	Full †
	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$	$\mathbb{1}(USD)$
$\mathbb{1}(Post\_Covid)$	0.0269**	0.00772**	0.0250***	0.0325***
	(0.0122)	(0.00375)	(0.00841)	(0.0121)
Controls	X	X	X	X
Firm FE	X	X	X	X
Observations	4,978	24,506	5,866	6,472
R-squared	0.751	0.924	0.875	0.666

Notes: Robust standard errors are in parantheses. *, **, *** denote significance at the 10, 5, and 1% levels, respectively. The sample in columns (1), (2) and (3) is restricted to issuance in five currencies, USD, EUR, JPY, CHF, GBP, in 2019 and 2020 (data retrieved in October 2020 from Dealogic). $\mathbb{1}(USD)$ is a dummy variable that takes the value 1 if the currency of the issued bond is the dollar. $\mathbb{1}(Post\_Covid)$ is a dummy that is one if the issue date is after 1 April 2020. Controls include the winsorized maturity at 5%, the size of the issuance and a dummy variable for the status of investment-grade status of the bond. All columns include a firm fixed effect. B refers to the sample of banks, $NBFI$ refers to the sample of non-bank financials and NFC refers to the sample of non-financial corporations. Column (4) includes all these sectors. The † in column (4) means that observations are limited to dollar and euro issuance by firms outside the United States and the Euroarea.

7 Conclusion and Policy Implications

Motivated by two facts, namely the dollar's dominant international role in debt markets and the fall and the subsequent rise of the dollar in these markets over the last two decades, we address two questions. First, of all the major international currencies, why is the dollar the dominant currency? Second, what explains the fall and the rise of the dollar?

We propose a “debt view” to explain the dollar’s dominant international role and provide broad empirical support for it. We develop a simple capital structure model in which firms optimally choose the currency composition of their debt. Independent of the lenders’ stochastic discount factor, borrowers behave as if they have a “CAPM discount factor,” whereby the debt currency choice of borrowers depends on how each currency co-moves with the firm’s stock value. In this sense, borrowers prefer debt issuance in the riskiest of international currencies. Both forward-looking and historical covariances suggest that the dollar fits this description better than all major currencies, especially for longer horizons. Moreover, the debt view can jointly explain the fall and the rise of the dollar in debt markets during the pre- and post-2008 and offer insights into the future of the dollar’s international role in the aftermath of the Covid-19 crisis.

The debt view is borrower-driven in contrast to the conventional view, which is investor-driven. The debt view can account for why firms issue in dollars despite the dollar not being the “safest” currency. It can also account for why the dollar has higher nominal interest rates and why the dollar’s international role was cemented following its depreciation against major currencies in the 1970s after the Bretton Woods. Moreover, once firms prefer to issue dollar debt, it might be possible to partially explain the dollar’s role in the rest of the international monetary and financial system, for example, why firms with dollar debt would prefer to invoice trade in dollars, or why a central bank would accumulate dollar reserves, among others. However, we leave this broader analysis of the dollar’s international role to future work.

Our results have some policy implications. First, it is commonly believed that an exchange rate depreciation could help an economy in downturns mainly through its effect on the terms of trade. Our results imply that exchange rate depreciation could also help an economy by reducing the probability of default of indebted firms. Second, our results imply that if a country wishes to gain a dominant currency status for debt issuance, it is essential that that country's currency is not the "safest haven" currency and riskier than its counterparts. Moreover, an essential role for the central bank arises, which is not to have realized inflation undershoot inflation expectations in downturns, generating appreciation pressures for the currency. To that end, if the European Union wants the euro as a dominant international currency, the debt view would suggest that this could be possible if inflation were more countercyclical.

What do our results imply for the future of the dollar? Many explanations of the dollar's dominant role in the international monetary system feature arguments like inertia, size, network externalities, and market liquidity. All these arguments suggest that changes in the dominance status of a currency occur very slowly. By contrast, our results indicate that the dollar can lose its dominance if the expectations about the risk properties of the dollar and other currencies change. As this relies on the beliefs of market participants, changes might occur abruptly. Our evidence from quarterly regressions suggests that this is a relevant channel.

This paper fits into a broader research agenda that aims to study the use of various currencies in different parts of the economy through the lens of their risk properties. Our model can be extended in multiple directions. First, addressing the interactions between the dollar's role in trade, banking and finance may shed important light on how debt issued in a dominant currency could affect other parts of the economy through the lens of the debt view. Second, modeling the demand for safe assets would help understand the role of the dollar for financial intermediation and household balance sheets and firms issuing debt jointly. We leave these questions for future research.

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Internet Appendix

A Appendix to Section 1: Theory

Proposition A.1 *Suppose that firms have a possibility of hedging foreign exchange risk by acquiring* $h_t \geq 0$ *units of a financial derivative contract with a payoff of* $X_{t+1} \geq 0$ *and a price of* $E_t[M_{t,t+1}X_{t+1}]$ *to be paid at time* t*. The firm always chooses* $h_t = 0$.

The intuition behind this result is straightforward. Hedging effectively plays a role of investment, and the firm only gets the payoff X_{t+1} from this investment in good (survival) states, while paying the market price at time t to get the payoff in all states. Thus, hedging is just a transfer of funds from shareholders to debt-holders, and firms optimally decide to minimize this transfer.[44]

Proof of Proposition A.1. The maximization problem is

$$\max_{h_t} \Bigg\{ - E_t[M_{t,t+1}X_{t+1}]h_t \\ + E_t \left[M_{t,t+1} \int_{\Omega_{t+1}Z_{t+1} > \mathcal{B}_{t+1}(B_t) - h_t(1-\tau)X_{t+1}} (\Omega_{t+1}Z_{t+1} - \mathcal{B}_{t+1}(B_t) + h_t(1-\tau)X_{t+1})\phi(Z_{t+1})dZ_{t+1} \right] \Bigg\}.$$

[44] There is ample evidence that firms often choose not to hedge their foreign currency risk. See, for example, Bodnár (2006) who shows that only 4% of Hungarian firms with foreign currency debt hedge their currency risk exposure. Furthermore, according to Salomao and Varela (2020): "data from the Central Bank of Peru reveals that only 6% of firms borrowing in foreign currency employ financial instruments to hedge the exchange rate risk, and a similar number is found in Brazil." Du and Schreger (2017) also provide evidence that firms do not fully hedge their currency risk exposures. See also Niepmann and Schmidt-Eisenlohr (2019), Bruno and Shin (2017). While it is known that costly external financing makes hedging optimal (see, for example, Froot, Scharfstein and Stein (1993) and Hugonnier, Malamud and Morellec (2015)), Rampini, Sufi and Viswanathan (2014) show both theoretically and empirically that, in fact, more financially constrained firms hedge less.

The derivative of this objective function with respect to h_t is given by

$$-E_t[M_{t,t+1}X_{t+1}] \;+\; (1-\tau)E_t\left[M_{t,t+1}X_{t+1}\left(1-\Phi\left(\frac{\mathcal{B}_{t+1}(B_t)-h_t(1-\tau)X_{t+1}}{\Omega_{t+1}}\right)\right)\right] \;<\; 0\,,$$

and hence $h_t = 0$ is optimal. Q.E.D.

Proof of Theorem 1.1. Firm's problem is to maximize

$$\sum_j E_t\left[M_{t,t+1}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\right]B_{j,t}(1-q(j))$$
$$+\;E_t\left[M_{t,t+1}\left[-\mathcal{B}_{t+1}(B_t)\left(1-\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\right)+\Omega_{t+1}\ell(\ell+1)^{-1}\left(1-\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell+1}\right)\right]\right]$$

Differentiating, we get from the standard Kuhn-Tucker conditions that borrowing only in dollars is optimal if and only if

$$E_t\left[M_{t,t+1}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\right](1-q(j))$$
$$+\,E_t\left[M_{t,t+1}\left[\left(-\ell(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell-1}\Omega_{t+1}^{-1}\right)(1+c)\mathcal{E}_{\$,i,t+1}(1+c(1-\tau))\mathcal{E}_{j,i,t+1}\right]\right]B_{\$,t}(1-q(\$))$$
$$-\,(1+c(1-\tau))E_t\left[M_{t,t+1}\mathcal{E}_{j,i,t+1}\right]$$
$$+\,E_t\Bigg[M_{t,t+1}(\ell+1)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,i,t+1}$$
$$-\,\ell\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,i,t+1}\Bigg]\leq 0$$

for all j with the identity for $j = \$$. This inequality can be rewritten as

$$
\begin{aligned}
&E_t[M_{t,t+1}\mathcal{E}_{j,i,t+1}]((1-q(j))(1+c)-(1+c(1-\tau))) \\
\leq\ & E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\mathcal{E}_{j,i,t+1}\right]((1-\rho)(1+c)[(1-q(j))+\ell(1-q(\$))]-(1+c(1-\tau)))
\end{aligned}
$$

At the same time, for the dollar debt we get

$$
\begin{aligned}
&E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]((1-q(\$))(1+c)-(1+c(1-\tau))) \\
=\ & E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\mathcal{E}_{\$,i,t+1}\right]((1+\ell)(1-\rho)(1+c)(1-q(\$))-(1+c(1-\tau)))
\end{aligned}
$$

implying that

$$
B_{\$,t}(1+c(1-\tau)) \ = \ \left(\frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\mathcal{E}_{\$,i,t+1}\Omega_{t+1}^{-\ell}\right]}\right)^{\ell^{-1}},
$$

and we get the Kuhn-Tucker conditions

$$
\frac{\bar{q}(j,\$)}{\bar{q}(\$)}\frac{E_t[M_{t,t+1}\mathcal{E}_{j,i,t+1}]}{E_t\left[M_{t,t+1}\Omega_{t+1}^{-\ell}\mathcal{E}_{j,i,t+1}\right]} \ \leq \ \frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\mathcal{E}_{\$,i,t+1}\Omega_{t+1}^{-\ell}\right]},
$$

and the claim follows. Q.E.D.

The next lemma formulates the optimality in terms of the pricing kernel $M_{t,t+1}^k = M_{t,t+1}^{\$}\mathcal{E}_{k,t,t+1}$ in a different currency k.

Lemma A.2 *Issuing in dollars is optimal if and only if*

$$
\frac{\bar{q}(j,\$)}{\bar{q}(\$)}\frac{E_t[M_{t,t+1}^k\mathcal{E}_{j,k,t+1}]}{E_t\left[M_{t,t+1}^k\tilde{\Omega}_{t+1}^{-\ell}\mathcal{E}_{j,k,t+1}\mathcal{E}_{\$,k,t+1}^{\ell}\right]} \ \leq \ \frac{E_t[M_{t,t+1}^k\mathcal{E}_{\$,k,t+1}]}{E_t\left[M_{t,t+1}^k\tilde{\Omega}_{t+1}^{-\ell}\mathcal{E}_{\$,k,t+1}^{1+\ell}\right]}
$$

for all j.

Proof. The currency-k price of debt denominated in currency j satisfies

$$\delta^j(B_t,k) = E_t[M^k_{t,t+1}\,(1-(1-\rho)\Phi(\Psi_{t+1}(B_t)))\,(1+c)\mathcal{E}_{j,t+1}/\mathcal{E}_{k,t+1}]$$

where $M^k_{t,t+1} = M^{\$}_{t,t+1}\mathcal{E}_{k,t,t+1}$ is the pricing kernel in currency k.

Let now $\tilde{V}_t \; = \; V_t/\mathcal{E}_{k,t}$ be the firm equity value in dollars and similarly $\tilde{\Omega} = \Omega/\mathcal{E}_k$ and $\tilde{\mathcal{B}}_{t+1} = \mathcal{B}_{t+1}/\mathcal{E}_{k,t+1}$ is the debt payoff denominated. Then,

$$\tilde{V}_t \;=\; V_t/\mathcal{E}_{k,t} \;=\; \tilde{\Omega}_t Z_t + \max_{B_t}\left\{\sum_{j=1}^{N}\delta^j(B_t,k)B_{j,t}(1-q(j))+E_t[M^k_{t,t+1}\max\{\tilde{V}_{t+1}-\tilde{\mathcal{B}}_{t+1}(B_t),0\}]\right\}$$

and thus nothing changes. Thus, repeating the above argument, dollar debt is optimal if and only if

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)}\frac{E_t[M^k_{t,t+1}\mathcal{E}_{j,k,t+1}]}{E_t\left[M^k_{t,t+1}\tilde{\Omega}^{-\ell}_{t+1}\mathcal{E}_{j,k,t+1}\mathcal{E}^{\ell}_{\$,k,t+1}\right]} \;\le\; \frac{E_t[M^k_{t,t+1}\mathcal{E}_{\$,k,t+1}]}{E_t\left[M^k_{t,t+1}\tilde{\Omega}^{-\ell}_{t+1}\mathcal{E}^{1+\ell}_{\$,k,t+1}\right]}$$

Q.E.D.

Proof of Theorem 1.3. follows from the following known result.

Lemma A.3 *Suppose that f, g are monotone decreasing and bounded. Then,*

$$\mathrm{Cov}_t(f(X), g(X)) \;\ge\; 0$$

for any bounded random variable X.

We need to compute

$$IRP_t \;=\; \frac{e^{r_t}\mathrm{Cov}_t(M_{t,t+1},\mathcal{P}_{i,t,t+1})}{E_t[\mathcal{P}_{i,t+1}]}\,.$$

For simplicity, we will assume that all idiosyncratic shocks are identically zero. Define $\tilde{a}_t = -\log S_t$. Our goal is to prove that

$$\begin{aligned} IRP_t + 1 &= \frac{E_t[M_{t,t+1}\mathcal{P}_{i,t,t+1}]}{E_t[M_{t,t+1}]E_t[\mathcal{P}_{i,t,t+1}]} \\ &= \frac{E_t[e^{(\phi+\gamma)\tilde{a}_{t+1}}]}{E_t[e^{\gamma\tilde{a}_{t+1}}]E_t[e^{\phi\tilde{a}_{t+1}}]} \end{aligned}$$

is monotone increasing in ϕ. We have

$$\frac{\partial}{\partial\phi}\log(IRP_t(\phi) \; + \; 1) \; = \; \frac{E_t[e^{\tilde{a}_{t+1}(\phi+\gamma)}\tilde{a}_{t+1}]}{E_t[e^{\tilde{a}_{t+1}(\phi+\gamma)}]} \; - \; \frac{E_t[e^{\tilde{a}_{t+1}\phi}\tilde{a}_{t+1}]}{E_t[e^{\tilde{a}_{t+1}\phi}]}$$

Making a change of measure $d\tilde{P} \; = \; e^{\tilde{a}_{t+1}\phi}/E_t[e^{\tilde{a}_{t+1}\phi}]$, we can rewrite the required inequality as

$$\frac{\tilde{E}_t[e^{\gamma\tilde{a}_{t+1}}\tilde{a}_{t+1}]}{\tilde{E}_t[e^{\gamma\tilde{a}_{t+1}}]} \; > \; \tilde{E}_t[\tilde{a}_{t+1}]\,,$$

which is a direct consequence of Lemma A.3. Q.E.D.

B Appendix to Section 2: Evidence from Forward-Looking Measures

Table 7: QRP, IRP, debt currency choice: Sample restricted to banks

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	Banks	Banks	Banks	Banks	Banks	Banks
	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}
$QRP^{2Y}_{€/\$,t}$	-3.369***	-1.512*	-3.039***			
	(0.403)	(0.761)	(0.467)			
$IRP^{2Y}_{\$,t}$				-0.00366	0.000598	0.0487**
				(0.0204)	(0.0258)	(0.0211)
$IRP^{2Y}_{€,t}$				-0.192***	-0.203***	-0.163***
				(0.0319)	(0.0513)	(0.0302)
Trend		X			X	
Control			X			X
Period	09q4-15q3	09q4-15q3	09q4-15q3	99q1-19q4	99q1-19q4	99q1-19q4
Observations	24	24	24	84	84	84
R-squared	0.644	0.708	0.710	0.284	0.285	0.478

Notes: Robust standard errors are shown in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. Debt issuance data includes only banks. Latest observed values of $QRP^{2Y}_{€/\$,t}$, $IRP^{2Y}_{\$,t}$ and $IRP^{2Y}_{€,t}$ in a given quarter are used. $QRP^{2Y}_{€/\$,t}$ data come from Kremens and Martin (2019), and $IRP^{2Y}_{\$,t}$ and $IRP^{2Y}_{€,t}$ come from Hördahl and Tristani (2014). Trend refers to a linear time trend and control refers to the inclusion of total issuance as a control variable.

Table 8: QRP, IRP, debt currency choice: Sample restricted to non-banks

	(1)	(2)	(3)	(4)	(5)	(6)
Sample:	Non-Banks	Non-Banks	Non-Banks	Non-Banks	Non-Banks	Non-Banks
	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}	USD_t^{shr}
$QRP_{€/\$,t}^{2Y}$	-2.750***	-1.759	-2.859***			
	(0.470)	(1.174)	(0.640)			
$IRP_{\$,t}^{2Y}$				-0.000896	-0.0124	0.0232
				(0.0207)	(0.0278)	(0.0252)
$IRP_{€,t}^{2Y}$				-0.196***	-0.166***	-0.203***
				(0.0253)	(0.0373)	(0.0226)
Trend		X			X	
Control			X			X
Period	09q4-15q3	09q4-15q3	09q4-15q3	99q1-19q4	99q1-19q4	99q1-19q4
Observations	24	24	24	84	84	84
R-squared	0.509	0.531	0.512	0.319	0.325	0.379

Notes: Robust standard errors are shown in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. Debt issuance data includes only non-banks. Latest observed values of $QRP_{€/\$,t}^{2Y}$, $IRP_{\$,t}^{2Y}$ and $IRP_{€,t}^{2Y}$ in a given quarter are used. $QRP_{€/\$,t}^{2Y}$ data come from Kremens and Martin (2019), and $IRP_{\$,t}^{2Y}$ and $IRP_{€,t}^{2Y}$ come from Hördahl and Tristani (2014). Trend refers to a linear time trend and control refers to the inclusion of total issuance as a control variable.

C Appendix to Section 3: Evidence from Backward-Looking Measures

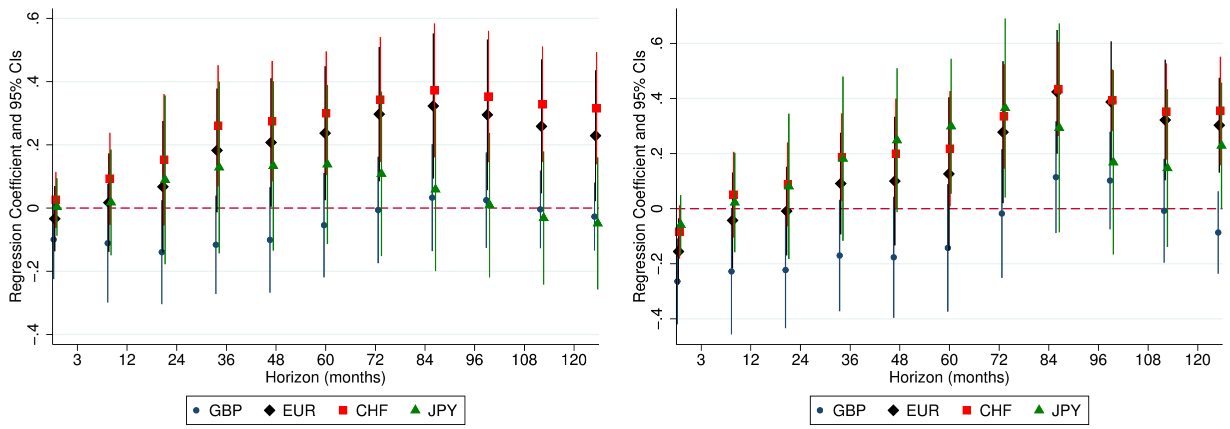
In order to complement the results from Section 3, in this appendix, we first redo the regressions in Section 3.1, using bilateral exchange rates as opposed to the dollar index. We show that the results are qualitatively similar, especially in terms of the patterns of short-term and longer-term covariances. Next, guided by these results, we compare the international debt share of the yen and the pound, currencies of countries that command a roughly similar share of the world economy. We show that the share of the pound and the yen in international debt markets behave in line with the risk properties of these currencies, in line with the mechanisms in our theory. We repeat the VAR analysis with the MSCI World Index instead of S&P 500 and show that our results are similar and even stronger. Finally, we report the results of the simple regressions between the dollar index and stock market indices with non-overlapping observations.

C.1 Backward-looking results with bilateral exchange rates

In this section, we provide the results for the same regressions as in Section 3.1, but using bilateral exchange rates for the dollar against four other major currencies. As Figure 8 shows, the dominant currency condition (2) holds empirically with currency j being the euro (EUR), the yen (JPY), or the Swiss franc (CHF). The only exception is British pound (GBP), for which our empirical proxy estimates in Figure 8 for the covariance in (2) have a negative sign. However, these covariance estimates are statistically insignificantly different from zero at the horizons of average debt maturity of firms. Thus, even absent differences in issuance costs, firms would strictly prefer issuing debt denominated in dollars, even if they could issue in EUR, JPY, or CHF. And even a slight difference in issuance costs favouring dollar to GBP would also make dollar immediately dominate over GBP.

Figure 8: The betas of the bilateral exchange rate of the dollar against major currencies with respect to stock market indices

S&P 500 Index *MSCI AC World Index*

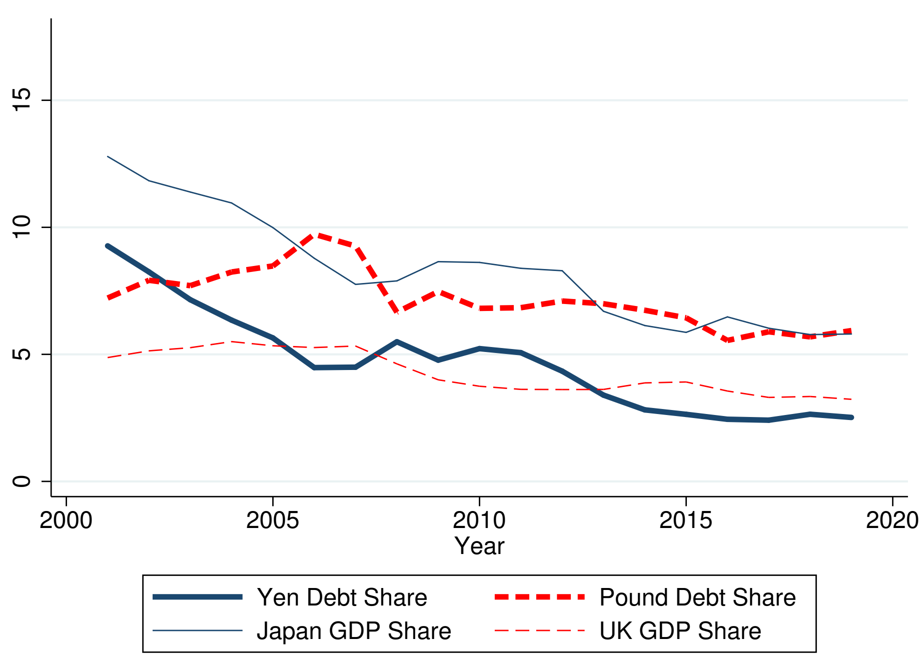


Notes: The graph on the left-hand side herein reports the regression coefficients β_h from the regressions (4) using the S&P 500 index. The graph on the right-hand side reports the regression coefficients from the regressions (4) using the MSCI AC World Index. The dots show the corresponding values of the β_h coefficients, while the lines show the 95% confidence intervals for these coefficients. Standard errors are corrected using the Newey-West procedure with the number of lags being equal to the horizon h of returns for each respective regression. The sample period for the S&P 500 goes from January 1973-December 2019. The sample period for the MSCI AC World Index goes from January 1988-December 2019 since data are only available starting from 1988.

C.2 Yen vs. Pound

As we show in Section C.1, the risk properties of the dollar alone can explain why the dollar dominates the euro, the yen and the Swiss franc in the sense of Theorem 1.1. One notable case is the pound: By Figure 8, the pound has favorable risk properties for debt issuers compared to most of the other major currencies. In reality, there are many reasons why the pound may not be the most obvious competitor to the dollar, such as differences in the size of the economies, lower issuance costs for the dollar etc. However, it is reasonable to compare the dynamics of debt issuance in GBP to that in JPY, since Japan and the Great Britain have similar size in the world economy. In this case, Figure 8 shows that (2) holds empirically if we replace \$ with GBP and choose j=JPY. Hence, firms should strictly prefer issuing in GBP to issuing in JPY. Figure 9 is in line with this prediction of our model. Indeed, surprisingly, despite the slightly larger share of Japan in the world economy and lower nominal interest rates and inflation in Japan, the share of pound-denominated debt is higher than the share of yen-denominated debt.

Figure 9: The yen versus the pound



Source: BIS, IMF WEO, authors' calculations

C.3 VAR Results using the MSCI World Index

In this subsection, we redo the VAR analysis conducted in Section 3 with the MSCI All Country World Index instead of the S&P 500. Note that due to data availability, our sample period runs only from 1988 to 2019. We present the results of the estimation of the VAR(2) model as well the cumulative impulse response functions to a negative shock to the MSCI All Country World Index below.

Table 9: A VAR(2) model of the MSCI World Index and the FRED dollar index

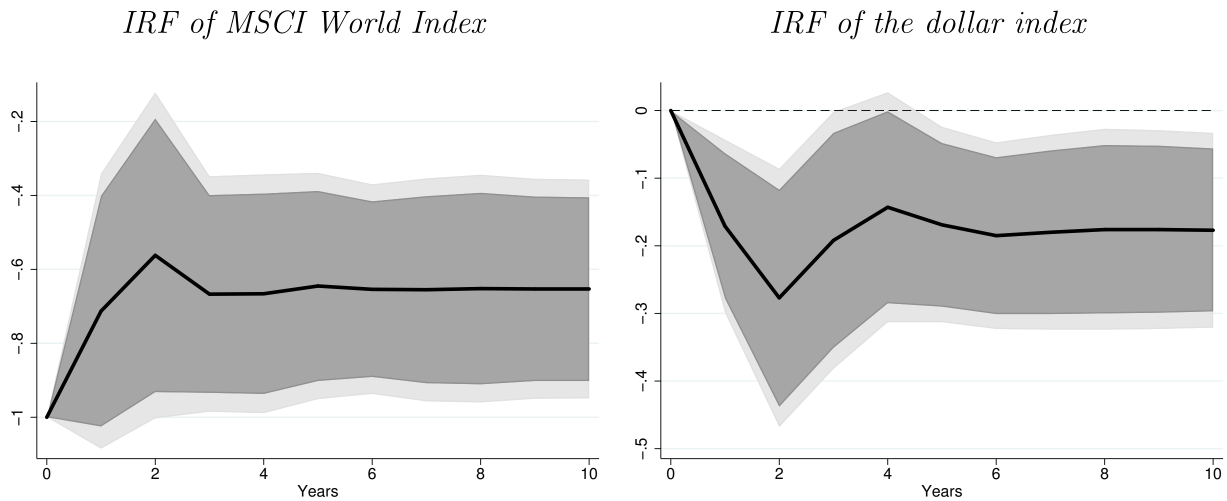
	(1)	(2)
	$Ret\_MSCI_{t-1,t}$	$Ret\_USD_{t-1,t}$
$Ret\_MSCI_{t-2,t-1}$	-0.287	0.171**
	(0.208)	(0.0716)
$Ret\_MSCI_{t-3,t-2}$	-0.236	0.118
	(0.220)	(0.0756)
$Ret\_USD_{t-2,t-1}$	0.0140	0.219
	(0.536)	(0.184)
$Ret\_USD_{t-3,t-2}$	-0.0452	-0.286
	(0.519)	(0.179)
Observations	30	30
R-squared	0.1065	0.3433

Notes: Standard errors that are adjusted for small-sample degrees of freedom in parantheses. *, **, *** denote significance at the 10, 5, and 1% levels, respectively. The coefficients are from a VAR(2) model of the yearly returns on the MSCI World Index and the FRED dollar index against major currencies (DTWEXM) between 1988 and 2019. The variance-covariance matrix for the error terms is estimated as: $\hat{\Sigma} = \begin{pmatrix} 0.0270 & \\ -0.0027 & 0.0032 \end{pmatrix}$

C.4 Simple Regressions Using Non-Overlapping Observations

In this appendix, we repeat the exercise in Section 3.1, but with non-overlapping observations, for example we using data from January 1973, January 1978... etc to calculate five-year returns, and proceed similarly for other horizons, without any overlap between observations.

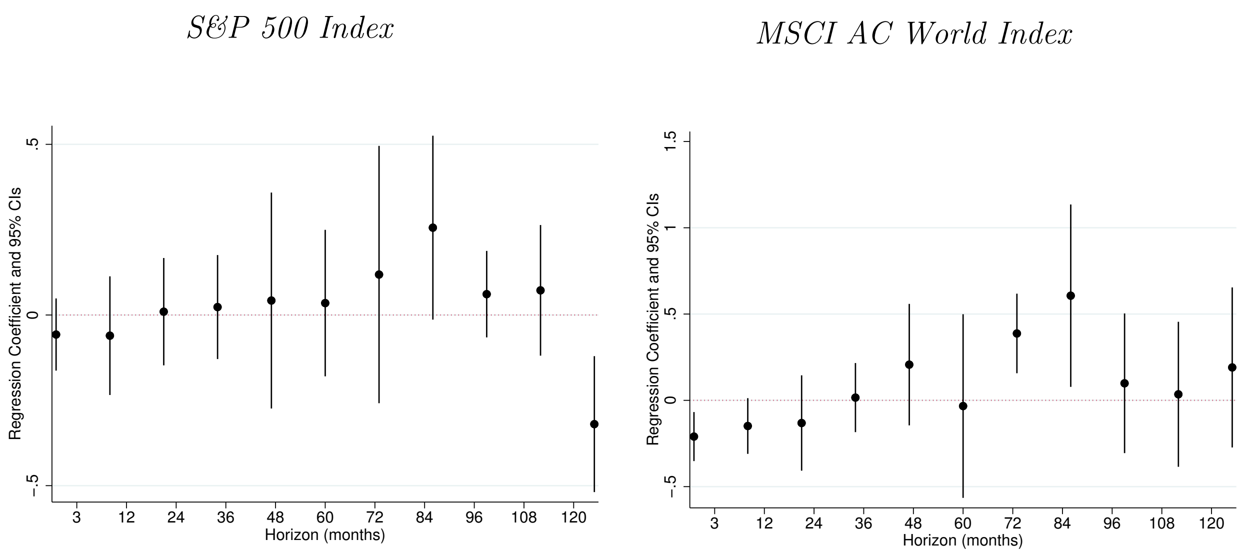
Figure 10: Cumulative Impulse Response Functions of a Shock to MSCI World Index



Source: Datastream, FRED, authors' calculations.
Notes: Figures show the cumulative impulse response functions of a negative 1 ppt shock to the MSCI World Index based on the estimates of a VAR(2) model of the yearly returns on the MSCI World Index and the FRED dollar index against major currencies (DTWEXM) between 1988 and 2019, reported in Table 9. The lines in each graph represent the cumulative impulse response functions. The darker shaded areas represent the 90% confidence intervals, while the lighter shaded areas represent the 95% confidence intervals.

Since our sample period is not sufficiently large, this approach necessarily leads to very small sample sizes. For example, since we have 46 years of data, five-year horizon only allows for 9 observations. We nevertheless report the results, for S&P 500 using a sample between 1973 and 2019, and for the MSCI World index, using a sample between 1988 and 2019, below. While in most cases, the results are not statistically significant, they follow a similar pattern that we have shown in Section 3.1.

Figure 11: The betas of the USD index returns with respect to stock market indices using non-overlapping observations



Notes: The graphs report the regression coefficients β_h from the regressions (4) using non-overlapping samples across different horizons, h. The round dots and the corresponding solid lines in both panels represent the point estimates, β_h, and 95% confidence intervals obtained using robust standard errors. The left-hand panel uses a sample period between January 1973 and December 2019. The right-hand panel uses a sample period between January 1988 and December 2019.

D Appendix to Section 5: Dynamic Capital Structure

Firm's problem is to maximize

$$W_1(B_0) \;=\; \sum_j E_1\left[M_{1,2}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,2}\right]\right]B_{j,1}(1-q_c(j))$$
$$+\;E_1\left[M_{1,2}\left[-\mathcal{B}_2(B_1+B_0)\left(1-\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}\right)+\Omega_2\ell(\ell+1)^{-1}\left(1-\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell+1}\right)\right]\right]$$

Differentiating, we get from the standard Kuhn-Tucker conditions that borrowing only in dollars is optimal if and only if

$$E_1\left[M_{1,2}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,2}\right]\right](1-q_c)$$
$$+E_1\left[M_{1,2}\left[\left(-\ell(1-\rho)\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell-1}\Omega_2^{-1}\right)(1+c)(1+c(1-\tau))\mathcal{E}_{j,2}\right]\right]B_{\$,1}(1-q_c(\$))$$
$$-(1+c(1-\tau))E_1\left[M_{1,2}\mathcal{E}_{j,2}\right]$$
$$+E_1\left[M_{1,2}(\ell+1)\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,2}\right.$$
$$\left.-\ell\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,2}\right]\le 0$$

for all j with the identity for $j=\$$. This inequality can be rewritten as

$$E_1[M_{1,2}\mathcal{E}_{j,2}]((1-q_c)(1+c)-(1+c(1-\tau)))$$
$$\le\;E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell}\mathcal{E}_{j,2}\right]((1-\rho)(1+c)[(1-q_c)]-(1+c(1-\tau)))$$
$$+\;(1-q_c)\ell(1-\rho)(1+c)(1+c(1-\tau))E_1\left[M_{1,2}\left[\left(\left(\frac{\mathcal{B}_2(B_1+B_0)}{\Omega_2}\right)^{\ell-1}\Omega_2^{-1}\right)\mathcal{E}_{j,2}\right]\right]B_{\$,1}$$

At the same time, for dollar debt we get

$$
\begin{aligned}
&E_1[M_{1,2}]((1-q_c(\$))(1+c)-(1+c(1-\tau))) \\
=\ &E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_0)+(1+c(1-\tau))B_{1,\$}}{\Omega_2}\right)^{\ell}\right]((1-\rho)(1+c)[(1-q)]-(1+c(1-\tau))) \\
+\ &(1-q)\ell(1-\rho)(1+c)E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_0)+(1+c(1-\tau))B_{1,\$}}{\Omega_2}\right)^{\ell}\frac{(1+c(1-\tau))B_{\$,1}}{\mathcal{B}_2(B_0)+(1+c(1-\tau))B_{1,\$}}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
&E_1[M_{1,2}\mathcal{E}_{j,2}]((1-q)(1+c)-(1+c(1-\tau))) \\
\leq\ &E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_0)+(1+c(1-\tau))B_{1,\$}}{\Omega_2}\right)^{\ell}\mathcal{E}_{j,2}\right]((1-\rho)(1+c)[(1-q)]-(1+c(1-\tau))) \\
+\ &(1-q)\ell(1-\rho)(1+c)E_1\left[M_{1,2}\left(\frac{\mathcal{B}_2(B_0)+(1+c(1-\tau))B_{1,\$}}{\Omega_2}\right)^{\ell}\frac{(1+c(1-\tau))B_{\$,1}}{\mathcal{B}_2(B_0)+(1+c(1-\tau))B_{1,\$}}\mathcal{E}_{j,2}\right]
\end{aligned}
$$

Now, we will just be verifying the Kuhn-Tucker conditions at time zero when B_0 is purely in US dollars. In this case,

$$
\begin{aligned}
&E_1[M_{1,2}\mathcal{E}_{j,2}]((1-q)(1+c)-(1+c(1-\tau))) \\
\leq\ &E_1\left[M_{1,2}\left(\frac{(1+c(1-\tau))(B_{0,\$}+B_{1,\$})}{\Omega_2}\right)^{\ell}\mathcal{E}_{j,2}\right]((1-\rho)(1+c)[(1-q)]-(1+c(1-\tau))) \\
+\ &(1-q)\ell(1-\rho)(1+c)E_1\left[M_{1,2}\left(\frac{(1+c(1-\tau))(B_{0,\$}+B_{1,\$})}{\Omega_2}\right)^{\ell}\frac{B_{\$,1}}{B_{0,\$}+B_{1,\$}}\mathcal{E}_{j,2}\right]
\end{aligned}
$$

while the dollar debt satisfies

$$
\begin{aligned}
& E_1[M_{1,2}]((1-q)(1+c)-(1+c(1-\tau))) \\
=\ & E_1\left[M_{1,2}\left(\frac{(1+c(1-\tau))(B_{0,\$}+B_{1,\$})}{\Omega_2}\right)^{\ell}\right]((1-\rho)(1+c)[(1-q)]-(1+c(1-\tau))) \\
+\ & (1-q)\ell(1-\rho)(1+c)E_1\left[M_{1,2}\left(\frac{(1+c(1-\tau))(B_{0,\$}+B_{1,\$})}{\Omega_2}\right)^{\ell}\frac{B_{\$,1}}{B_{0,\$}+B_{1,\$}}\right]
\end{aligned} \tag{6}
$$

Dividing, we get

$$
\frac{E_1[M_{1,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]} \ = \ x(B_{0,\$}+B_{1,\$})^{\ell}+y(B_{0,\$}+B_{1,\$})^{\ell-1}B_{1,\$}
$$

whereas

$$
\frac{E_1[M_{1,2}\mathcal{E}_{j,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}]} \ \leq \ \frac{E_1[M_{1,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]}
$$

so the condition is still the same. Here, $x, y > 0$ (we assume that $x > 0$) are constants. This defines $B_{1,\$} = F(X_1, B_0)$. However, to derive the first order conditions, we will need to compute derivatives of F with respect to other debt components. To this end, we need to differentiate the implicit equation

$$
\begin{aligned}
& \varepsilon_1 E_1[M_{1,2}] \\
& = \ xE_1\left[M_{1,2}\left(\sum_j \mathcal{E}_{j,2}B_{j,0}+B_{1,\$}\right)^{\ell}\Omega_2^{-\ell}\right] \\
& + E_1\left[M_{1,2}\Omega_2^{-\ell}\left(\sum_j \mathcal{E}_{j,2}B_{j,0}+B_{1,\$}\right)^{\ell-1}B_{\$,1}^{*}\right]
\end{aligned}
$$

Differentiating this identify with respect to $B^*_{j,0}$, we get

$$0 = x\ell E_1[M_{1,2}\Omega_2^{-\ell}(B_{0,\$}+B_{1,\$})^{\ell-1}(\mathcal{E}_{j,2}+B'_{1,\$})] \;+\; E_1\left[M_{1,2}\Omega_2^{-\ell}\left(B_{0,\$}+B_{1,\$}\right)^{\ell-1}\right]B'_{\$,1}$$
$$+(\ell-1)E_1[M_{1,2}\Omega_2^{-\ell}(B_{0,\$}+B_{1,\$})^{\ell-2}(\mathcal{E}_{j,2}+B'_{1,\$})]B_{\$,1}$$

implying that

$$\frac{\partial B_{\$,1}}{\partial B_{j,0}} \;=\; -\frac{E_1[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}](x\ell(B_{0,\$}+B_{1,\$})^{\ell}+(\ell-1)(B_{0,\$}+B_{1,\$})^{\ell-1}B_{\$,1})}{E_1[M_{1,2}\Omega_2^{-\ell}](x\ell(B_{0,\$}+B_{1,\$})^{\ell}+[(\ell-1)B_{\$,1}(B_{0,\$}+B_{1,\$})^{\ell-1})+(B_{0,\$}+B_{1,\$})^{\ell}]}$$
$$=\; -\frac{E_1[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]}\frac{\ell\varepsilon_1 E_1[M_{1,2}]-E_1[M_{1,2}\Omega_2^{-\ell}](B_{0,\$}+B_{1,\$})^{\ell-1}B_{\$,1}}{\ell\varepsilon_1 E_1[M_{1,2}]+E_1[M_{1,2}\Omega_2^{-\ell}](B_{0,\$}+B_{1,\$})^{\ell-1}B_{\$,0}}$$

Now we can solve the time zero problem. The objective with respect to B_0 is

$$\max_{B_0}\left\{\sum_j \delta_j(B_0)B_{j,0} \;+\; E[M_{0,1}\max\{W_1(B_0,X_1Z_1)-\sum_j c_1 B_{j,0}(1-\tau)\mathcal{E}_{j,1},0\}]\right\}$$

where

$$\delta_{j,0}(B_0) \;=\; E[M_{0,1}(c_1\mathcal{E}_{j,1}+\delta_{j,1}(B_0+F(X_1,B_0)))\mathbf{1}_{W_1(B_0,X_1Z_1)-\sum_j c_1 B_{j,0}(1-\tau)\mathcal{E}_{j,1}>0}]$$

and where

$$W_1(B_0,X_1) \;=\; \delta_{\$,1}(B_0+F(X_1,B_0)))F(X_1,B_0)(1-q_c)$$
$$+E_1[\mathbf{1}_{\Omega_2>\mathcal{B}_2}M_{1,2}(\ell(\ell+1)^{-1}\Omega_2-\mathcal{B}_2(B_1+B_0))]$$
$$+\;(\ell+1)^{-1}E_1\left[\mathbf{1}_{\Omega_2>\mathcal{B}_2}M_{1,2}\Omega_2^{-\ell}(\mathcal{B}_2(B_1+B_0))^{\ell+1}\right]$$

where we have defined

$$\delta_{j,1}(B_0+F(X_1,B_0))) \;=\; E_1\left[M_{1,2}\mathbf{1}_{\Omega_2>\mathcal{B}_2}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_2(F(X_1,B_0)+B_0)}{\Omega_2}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,2}\right]\right]$$

Now, by assumption, $Z_1 \sim \ell y^{\ell-1}$ on $[0,1]$. First, we need to figure out the threshold $\Theta_1(X_1, B_0)$ for default at time $t = 1$. It is determine via

$$W_1(B_0, X_1\Theta_1) - \sum_j c_1 B_{j,0}(1-\tau)\mathcal{E}_{j,1} \;=\; 0\,.$$

Substituting, we get the following equation for Θ_1 evaluated as $B = B_{\$,0}$:

$$\begin{aligned}
&\delta_{\$,1}(B_0 + B_{\$,1}(\Theta_1), \Theta_1) B_{\$,1}(\Theta_1)(1-q_c) \\
&+ E_1[\mathbf{1}_{\Omega_2 > \mathcal{B}_2/\Theta_1} M_{1,2}(\ell(\ell+1)^{-1}\Omega_2\Theta_1 - (1 + c(1-\tau))(B_{\$,1}(\Theta_1) + B_0))] \\
&+ \;(\ell+1)^{-1}(1+c(1-\tau))^{\ell+1} E_1\left[M_{1,2}\mathbf{1}_{\Omega_2 > \mathcal{B}_2/\Theta_1}\Theta_1^{-\ell}\Omega_2^{-\ell}(B_{\$,1}(\Theta_1) + B_0)^{\ell+1}\right] \;=\; c_1 B_{\$,0}(1-\tau)\,.
\end{aligned}$$

We will impose a technical condition that at that threshold it is optimal not to issue any more debt (indeed, this is the default threshold, so it makes perfect sense). Hence, B_1 is not there at Θ_1 and now the equation for Θ_1 becomes much simpler. By assumption, there are always some states in which the firm survives at time $t = 2$. This is equivalent to $\min(\Omega_2)\Theta_1 > B_{\$,0}(1 + c(1-\tau))$. In this case,

$$\Theta_1\, E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] \;+\; B_{\$,0}^{\ell+1}\Theta_1^{-\ell}(\ell+1)^{-1}E_1[M_{1,2}\Omega_2^{-\ell}] \;=\; (1+c(1-\tau)+c_1(1-\tau))B_{\$,0}$$

and hence

$$\Theta_1 \;=\; \Theta_1^* B_{\$,0}$$

where

$$\Theta_1^*\, E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] \;+\; (\Theta_1^*)^{-\ell}(\ell+1)^{-1}E_1[M_{1,2}\Omega_2^{-\ell}] \;=\; (1+c(1-\tau)+c_1(1-\tau))$$

and this equation has a solution if and only if

$$\left(\frac{E_1[M_{1,2}\Omega_2]}{E_1[M_{1,2}\Omega_2^{-\ell}]}\right)^{-1/(\ell+1)} E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] \;+\; \left(\frac{E_1[M_{1,2}\Omega_2]}{E_1[M_{1,2}\Omega_2^{-\ell}]}\right)^{\ell/(\ell+1)} (\ell+1)^{-1}E_1[M_{1,2}\Omega_2^{-\ell}]$$
$$<\; (1+c(1-\tau)+c_1(1-\tau))$$

That is, default threshold is homogeneous in $B_{\$,0}$ when no new debt is issued at the default threshold. Let us now verify the technical condition: It is equivalent to

$$\frac{1+c(1-\tau)}{\min\Omega_2}\,E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] + \left(\frac{1+c(1-\tau)}{\min\Omega_2}\right)^{-\ell}(\ell+1)^{-1}E_1[M_{1,2}\Omega_2^{-\ell}] > 1+c(1-\tau)+c_1(1-\tau)$$

Now, we can integrate the idiosyncratic shock away:

$$\delta_{j,0}(B_0) \;=\; E[M_{0,1}c_1\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)] \;+\; E[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \delta_{j,1}(B_0+F(X_1q,B_0))\ell q^{\ell-1}dq]$$
$$=\; E[M_{0,1}c_1\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)]$$
$$+\; E[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 E_1\left[M_{1,2}\left[\left(1-(1-\rho)\left(\frac{(1+c(1-\tau))((F(X_1q,B_0)+B_0)}{\Omega_2 q}\right)^\ell\right)(1+c)\mathcal{E}_{j,2}\right]\right]\ell q^{\ell-1}dq]$$
$$=\; E[(M_{0,1}c_1\mathcal{E}_{j,1}+M_{0,2}(1+c)\mathcal{E}_{j,2})] - B_0^\ell E[(M_{0,1}c_1\mathcal{E}_{j,1}+M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^\ell]$$
$$-\,(1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\int_{B_0\Theta_1^*}^1 \mathcal{B}_2(q)^\ell q^{-1}dq]$$

We will also need

$$\bar{W}_1(B_0,X_1,Q) \;=\; \int_Q^1 W_1(B_0,X_1q)\ell q^{\ell-1}dq$$

and now we can finally write down the full value function:

$$\sum_j \delta_j(B_0)B_{j,0} \; - E[M_{0,1}\sum_j c_1 B_{j,0}(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)]$$
$$+ \; E[M_{0,1}\bar{W}_1(B_0,X_1,\Theta_1(X_1,B_0))]$$

Our objective is to verify the Kuhn tucker conditions at the pure dollar debt equilibrium. We thus need to verify (note that the terms with Θ_1' cancels out):

$$\delta_j(B_0)(1-q_c) \; + \; \frac{\partial}{\partial B_{j,0}}\delta_\$(B_0)B_{\$,0}(1-q_c) \; - E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)]$$
$$+ \; E[M_{0,1}\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0,X_1,\Theta_1(X_1,B_0))] < 0$$

for all $j \neq \$$. Now, we have

$$\frac{\partial}{\partial B_{j,0}}\delta_{\$}(B_0) \;=\; \frac{\partial}{\partial B_{j,0}}\Bigg(E[M_{0,1}c_1(1-\Theta_1(X_1,B_0))^{\ell}]$$

$$+\;E[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \delta_{\$,1}(B_0+F(X_1q,B_0))\ell q^{\ell-1}dq]\Bigg)$$

$$=-\;\ell E[M_{0,1}c_1\Theta_1(X_1,B_0)^{\ell-1}\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0)]$$

$$-\;\ell E[M_{0,1}\Theta_1(X_1,B_0)^{\ell-1}\delta_{\$,1}(B_0+F(X_1\Theta_1(X_1,B_0),B_0))\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0)]$$

$$+E[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \frac{\partial}{\partial B_{j,0}}\delta_{\$,1}(B_0+F(qX_1,B_0)))\ell q^{\ell-1}dq]$$

$$=\;-\ell E\Big[M_{0,1}\Theta_1^{\ell-1}\Big(c_1+\delta_{\$,1}(B_0+F(X_1\Theta_1(X_1,B_0),B_0))\Big)\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0)\Big]$$

$$-\;E\Big[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \ell(1-\rho)(1+c(1-\tau))^{\ell}$$

$$E_1\left[M_{1,2}q^{-\ell}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}}+\mathcal{E}_{j,2}\Big)\left(F(qX_1,B_0)+B_0\right)^{\ell-1}(1+c)\right]\ell q^{\ell-1}dq\Big]$$

$$=\;-\ell E\Big[M_{0,1}\Theta_1^{\ell-1}\Big(c_1+\delta_{\$,1}(B_0+F(X_1\Theta_1(X_1,B_0),B_0))\Big)\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0)\Big]$$

$$-\;E\Big[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \ell(1-\rho)(1+c(1-\tau))^{\ell}$$

$$E_1\left[M_{1,2}q^{-\ell}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}}+\mathcal{E}_{j,2}\Big)\left(F(qX_1,B_0)+B_0\right)^{\ell-1}(1+c)\right]\ell q^{\ell-1}dq\Big]$$

Furthermore, (recall that we are always evaluating the derivatives at $B_{k,0}=0$ for all $k \neq \$$)

$$\frac{\partial}{\partial B_{j,0}}\delta_{\$,1}(B_0+F(X_1,B_0))) \;=\; \frac{\partial}{\partial B_{j,0}}E_1\left[M_{1,2}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_2(F(X_1,B_0)+B_0)}{\Omega_2}\right)^{\ell}\right)(1+c)\right]\right]$$

$$=-\ell(1-\rho)(1+c(1-\tau))^{\ell}E_1\left[M_{1,2}\Omega_2^{-\ell}\Big(\frac{\partial B_{\$,1}}{\partial B_{j,0}}+\mathcal{E}_{k,2}\Big)\left(F(X_1,B_0)+B_0\right)^{\ell-1}(1+c)\right]$$

and

$$\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0) \;=\; \frac{c_1(1-\tau)\mathcal{E}_{j,1}}{W_{1,X}(B_0,X_1\Theta_1)X_1}$$

whereas

$$\frac{\partial}{\partial B_{j,0}}W_1(B_0,X_1) \;=\; \frac{\partial}{\partial B_{j,0}}\Bigg(\delta_{\$,1}(B_0+B_{\$,1})B_{\$,1}(1-q_c)$$

$$+\,E_1[M_{1,2}(\ell(\ell+1)^{-1}\Omega_2-\mathcal{B}_2(B_1+B_0))]$$

$$+\;(\ell+1)^{-1}E_1\left[M_{1,2}\Omega_2^{-\ell}(\mathcal{B}_2(B_1+B_0))^{\ell+1}\right]\Bigg)$$

$$=\;-\ell(1-\rho)(1+c(1-\tau))^{\ell}E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(1-q_c)$$

$$-\,(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}]+(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}+B_0)^{\ell}\mathcal{E}_{j,2}\right]$$

and

$$\frac{\partial}{\partial q}W_1(B_0,X_1q) \;=\; \frac{\partial}{\partial q}\Bigg(\delta_{\$,1}(B_0+B_{\$,1}(q),q)B_{\$,1}(q)(1-q_c)$$

$$+\,E_1[M_{1,2}(\ell(\ell+1)^{-1}\Omega_2q-(1+c(1-\tau))(B_{\$,1}(q)+B_0))]$$

$$+\;(\ell+1)^{-1}E_1\left[M_{1,2}q^{-\ell}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell+1}\right]\Bigg)$$

$$=\;\ell(1-\rho)(1+c(1-\tau))^{\ell}q^{-\ell-1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell}(1+c)\right]B_{\$,1}(q)(1-q_c)$$

$$+\,E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2]\;-\;\ell(\ell+1)^{-1}(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}q^{-\ell-1}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell+1}\right]$$

but at Θ_1 (since we assume that no new debt is issued at the default threshold)

$$\frac{\partial}{\partial q}W_1(B_0,X_1q)|_{q=\Theta_1} \;=\; E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2]$$

$$-\;\ell(\ell+1)^{-1}(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Theta_1^{-\ell-1}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell+1}\right]$$

Furthermore, we know

$$E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] \;+\; B_{\$,0}^{\ell+1}\Theta_1^{-\ell-1}(\ell+1)^{-1}E_1[M_{1,2}\Omega_2^{-\ell}] \;=\; (1+c(1-\tau)+c_1(1-\tau))B_{\$,0}\Theta_1^{-1}$$

and therefore we can rewrite

$$\begin{aligned}
&E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] \;-\; \ell(\ell+1)^{-1}(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Theta_1^{-\ell-1}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell+1}\right] \\
&= \; E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2] \;-\; \ell\Big((1+c(1-\tau)+c_1(1-\tau))B_{\$,0}\Theta_1^{-1} - E_1[M_{1,2}\ell(\ell+1)^{-1}\Omega_2]\Big) \\
&= \; \ell(E_1[M_{1,2}\Omega_2] - (1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})
\end{aligned}$$

and hence

$$\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0) \;=\; \frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2] - (1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})}$$

Note that, by the envelope condition, we just need to differentiate with respect to B_0, keeping $B_{1,\$}$ fixed.

Finally,

$$\begin{aligned}
&\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0,X_1,Q) \;=\; \int_Q^1 \frac{\partial}{\partial B_{j,0}}W_1(B_0,X_1q)\ell q^{\ell-1}dq \\
&= \; \int_Q^1 \ell\Bigg(- \ell(1-\rho)(1+c(1-\tau))^{\ell}q^{-\ell}E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(q)(1-q_c) \\
&- (1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}] + (1+c(1-\tau))^{\ell+1}q^{-\ell}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell}\mathcal{E}_{j,2}\right]\Bigg)q^{\ell-1}dq \\
&= \; -(1-Q^{\ell})(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}] \\
&+ \; \ell\int_Q^1 q^{-1}(1+c(1-\tau))^{\ell}\Bigg(- \ell(1-\rho)E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(q)(1-q_c) \\
&+ (1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell}\mathcal{E}_{j,2}\right]\Bigg)dq\,.
\end{aligned}$$

Thus,

$$
\begin{aligned}
&\frac{\partial}{\partial B_{j,0}}\delta_{\$}(B_0)\\
&= \; -\ell E\Big[M_{0,1}\Theta_1^{\ell-1}\Big(c_1+\delta_{\$,1}(B_0+F(X_1\Theta_1(X_1,B_0),B_0))\Big)\frac{\partial}{\partial B_{j,0}}\Theta_1(X_1,B_0)\Big]\\
&- \; E\Big[M_{0,1}\int_{\Theta_1(X_1,B_0)}^{1}\ell(1-\rho)(1+c(1-\tau))^{\ell}\\
&E_1\left[M_{1,2}q^{-\ell}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}}+\mathcal{E}_{j,2}\Big)\left(F(qX_1,B_0)+B_0\right)^{\ell-1}(1+c)\right]\ell q^{\ell-1}dq\Big]\\
&= \; -\ell E\Big[M_{0,1}\Theta_1^{\ell-1}\Big(c_1+\delta_{\$,1}(B_0+F(X_1\Theta_1(X_1,B_0),B_0))\Big)\\
&\times\frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2]-(1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})}\Big]\\
&- \; E\Big[M_{0,1}\int_{\Theta_1(X_1,B_0)}^{1}\ell(1-\rho)(1+c(1-\tau))^{\ell}\\
&E_1\left[M_{1,2}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}}+\mathcal{E}_{j,2}\Big)\left(F(qX_1,B_0)+B_0\right)^{\ell-1}(1+c)\right]\ell q^{-1}dq\Big]
\end{aligned}
$$

whereas

$$
\begin{aligned}
&E[M_{0,1}\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0,X_1,\Theta_1(X_1,B_0))]\\
&= \; E\Bigg[M_{0,1}\Bigg(-(1-\Theta_1^{\ell})(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}]\\
&+ \; \ell\int_{\Theta_1}^{1}q^{-1}(1+c(1-\tau))^{\ell}\Bigg(-\ell(1-\rho)E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(q)(1-q_c)\\
&+(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell}\mathcal{E}_{j,2}\right]\Bigg)dq\Bigg)\Bigg]
\end{aligned}
$$

To proceed further, we need to derive the first order approximation to the policy function $B_{\$,1}$ using (6) under the assumption that $((1-q)(1+c)-(1+c(1-\tau)))$ is small. In this case,

$$
\begin{aligned}
& E_1[M_{1,2}]((1-q)(1+c)-(1+c(1-\tau))) \\
&= E_1\left[M_{1,2}\left(\frac{(1+c(1-\tau))(B_{0,\$}+B_{1,\$})}{\Omega_2}\right)^{\ell}\right]((1-\rho)(1+c)[(1-q)]-(1+c(1-\tau))) \\
&+ (1-q)\ell(1-\rho)(1+c)E_1\left[M_{1,2}\left(\frac{(1+c(1-\tau))(B_{0,\$}+B_{1,\$})}{\Omega_2}\right)^{\ell}\frac{B_{\$,1}}{B_{0,\$}+B_{1,\$}}\right]
\end{aligned}
$$

Let

$$
\varepsilon_1 = \frac{(1-q)(1+c)-(1+c(1-\tau))}{(1-q)\ell(1-\rho)(1+c)(1+c(1-\tau))^{\ell}},\ \varepsilon_2 = \frac{(1-q)(1-\rho)(1+c)-(1+c(1-\tau))}{(1-q)\ell(1-\rho)(1+c)(1+c(1-\tau))^{\ell}}
$$

and assume they are small (this is indeed the case when τ is small). Then, the gains from debt issuance are small and hence debt is small. Thus, B_0 and B_1 are proportional to $\varepsilon_1^{1/\ell}$ and $B_{1,\$}(q)$ (when Ω_2 is multiplied by q) solves

$$
q^{\ell}\frac{E_1[M_{1,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]}\varepsilon_1 = (B_0+B_1)^{\ell-1}B_1 + x(B_0+B_1)^{\ell}
$$

We will rescale them and denote $B_t^* = B_t/\varepsilon_1^{1/\ell}$. Then,

$$
q^{\ell}\frac{E_1[M_{1,2}]}{E_1[M_{1,2}\Omega_2^{-\ell}]} = (B_0^*+B_1^*)^{\ell-1}B_1^* + x(B_0^*+B_1^*)^{\ell}
$$

where we assume that x is constant.

Now we can gather all the terms and rewrite the Kuhn-Tucker condition as

$$
\begin{aligned}
&\delta_j(B_0)(1-q_c) \;+\; \frac{\partial}{\partial B_{j,0}}\delta_\$(B_0)B_{\$,0}(1-q_c) \;- E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)] \\
&+\; E[M_{0,1}\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0,X_1,\Theta_1(X_1,B_0))] \\
&=\; \Bigg(E[(M_{0,1}c_1\mathcal{E}_{j,1}+M_{0,2}(1+c)\mathcal{E}_{j,2})] - B_0^\ell E[(M_{0,1}c_1\mathcal{E}_{j,1}+M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^\ell] \\
&-(1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\int_{B_0\Theta_1^*}^1 \mathcal{B}_2(q)^\ell q^{-1}dq]\Bigg)(1-q_c) \\
&+B_{\$,0}\Bigg(-\ell E\Big[M_{0,1}\Theta_1^{\ell-1}\Big(c_1+\delta_{\$,1}(B_0+F(X_1\Theta_1(X_1,B_0),B_0))\Big) \\
&\times\frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2]-(1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})}\Big] \\
&-\; E\Big[M_{0,1}\int_{\Theta_1(X_1,B_0)}^1 \ell(1-\rho)(1+c(1-\tau))^\ell \\
&E_1\left[M_{1,2}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}}+\mathcal{E}_{j,2}\Big)\left(F(qX_1,B_0)+B_0\right)^{\ell-1}(1+c)\right]\ell q^{-1}dq\Big]\Bigg)(1-q_c) \\
&-E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)] \\
&+E\Bigg[M_{0,1}\Bigg(-(1-\Theta_1^\ell)(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}] \\
&+\;\ell\int_{\Theta_1}^1 q^{-1}(1+c(1-\tau))^\ell\Bigg(-\ell(1-\rho)E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(q)(1-q_c) \\
&+(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^\ell\mathcal{E}_{j,2}\right]\Bigg)dq\Bigg)\Bigg]
\end{aligned}
$$

Now, since B_0 is small, we will only keep the highest order terms (those of order $B_{\$,0}^\ell$) and ignore the terms of the order $o(B^\ell_{\$,0})$. This gives

$$
\begin{aligned}
&\delta_j(B_0)(1-q_c) \;+\; \frac{\partial}{\partial B_{j,0}}\delta_\$(B_0)B_{\$,0}(1-q_c) \;- E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-\Theta_1(X_1,B_0)^\ell)] \\
&+\; E[M_{0,1}\frac{\partial}{\partial B_{j,0}}\bar{W}_1(B_0,X_1,\Theta_1(X_1,B_0))] \\
&=\; \Bigg(E[(M_{0,1}c_1\mathcal{E}_{j,1}+M_{0,2}(1+c)\mathcal{E}_{j,2})] - B_0^\ell E[(M_{0,1}c_1\mathcal{E}_{j,1}+M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^\ell] \\
&-(1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\int_{\Theta_1}^1 \mathcal{B}_2(q)^\ell q^{-1}dq]\Bigg)(1-q_c) \\
&+B_{\$,0}\Bigg(-\ell B_{\$,0}^{\ell-1}E\Big[M_{0,1}(\Theta_1^*)^{\ell-1}\Big(c_1+E_1[M_{1,2}(1+c)]\Big) \\
&\times\frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2]-(1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})}\Big] \\
&-\; E\Big[M_{0,1}\int_{\Theta_1}^1 \ell(1-\rho)(1+c(1-\tau))^\ell \\
&E_1\left[M_{1,2}\Omega_2^{-\ell}\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}}+\mathcal{E}_{j,2}\Big)\left(F(qX_1,B_0)+B_0\right)^{\ell-1}(1+c)\right]\ell q^{-1}dq\Big]\Bigg)(1-q_c) \\
&-E[M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}(1-B_{\$,0}^\ell(\Theta_1^*)^\ell)] \\
&+\Bigg(E\Bigg[M_{0,1}\Bigg(-(1-B_{\$,0}^\ell(\Theta_1^*)^\ell)(1+c(1-\tau))E_1[M_{1,2}\mathcal{E}_{j,2}] \\
&+\;\ell\int_{\Theta_1}^1 q^{-1}(1+c(1-\tau))^\ell\Bigg(-\ell(1-\rho)E_1\left[M_{1,2}\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(q)(1-q_c) \\
&+(1+c(1-\tau))^{\ell+1}E_1\left[M_{1,2}\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^\ell\mathcal{E}_{j,2}\right]\Bigg)dq\Bigg)\Bigg]\Bigg) \;+\; o(\varepsilon_1^{1/\ell})
\end{aligned}
$$

We can now regroup this as

$$
\begin{aligned}
& E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}c\mathcal{E}_{j,2})](\tau - q_c) \\
& + B_{\$,0}^{\ell}\Bigg(- E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^{\ell}](1-q_c) \\
& - \ell E\Bigg[M_{0,1}(\Theta_1^*)^{\ell-1}\Big(c_1 + E_1[M_{1,2}(1+c)]\Big) \times \frac{c_1(1-\tau)\mathcal{E}_{j,1}}{\ell(E_1[M_{1,2}\Omega_2] - (1+c(1-\tau)+c_1(1-\tau))(\Theta_1^*)^{-1})}\Bigg](1-q_c) \\
& + E[(M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1} + (1+c(1-\tau))M_{0,2}\mathcal{E}_{j,2})(\Theta_1^*)^{\ell}]\Bigg) \\
& + \int_{\Theta_1}^{1} q^{-1} \\
\Bigg(& - (1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\mathcal{B}_2(q)^{\ell}(1-q_c) \\
& - \ell^2 E[M_{0,2}(1-\rho)\Omega_2^{-\ell}(1+c(1-\tau))\Big(\frac{F(qX_1,B_0)}{\partial B_{j,0}} + \mathcal{E}_{j,2}\Big)\mathcal{B}_2(q)^{\ell-1}B_{\$,0}(1+c)(1-q_c) \\
& + \ell E\Bigg[M_{0,2}(1+c(1-\tau))^{\ell}\Bigg(- \ell(1-\rho)\left[\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q)+B_0)^{\ell-1}(1+c)\right]B_{\$,1}(q)(1-q_c) \\
& + (1+c(1-\tau))^{\ell+1}\left[\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell}\mathcal{E}_{j,2}\right]\Bigg)\Bigg)dq \;+\; o(\varepsilon_1^{1/\ell})
\end{aligned}
$$

Recalling that we assume that q, τ, ρ are all of the order as ε_1. Thus,

$$
-(1-\rho)E[M_{0,2}(1+c)\mathcal{E}_{j,2}\Omega_2^{-\ell}\ell\mathcal{B}_2(q)^{\ell}(1-q_c) + (1+c(1-\tau))^{\ell+1}\left[\Omega_2^{-\ell}(B_{\$,1}(q)+B_0)^{\ell}\mathcal{E}_{j,2}\right] \;=\; O(\varepsilon_1^2)
$$

and

$$
B_{\$,0}^{\ell}(-E[(M_{0,1}c_1\mathcal{E}_{j,1}+M_{0,2}(1+c)\mathcal{E}_{j,2})(\Theta_1^*)^{\ell}](1-q_c)+E[(M_{0,1}c_1(1-\tau)\mathcal{E}_{j,1}+(1+c(1-\tau))M_{0,2}\mathcal{E}_{j,2})(\Theta_1^*)^{\ell}]) = O(\varepsilon_1^2
$$

Thus, we are only left with

$$
\begin{aligned}
&E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}c\mathcal{E}_{j,2})](\tau - q_c) \\
&+ B_{\$,0}^{\ell}\Bigg(- \ell E\Bigg[M_{0,1}(\Theta_1^*)^{\ell-1}\frac{c_1(1-\tau)\mathcal{E}_{j,1}\Big(c_1 + E_1[M_{1,2}(1+c)]\Big)}{\ell(E_1[M_{1,2}\Omega_2] - (1 + c(1-\tau) + c_1(1-\tau))(\Theta_1^*)^{-1})}\Bigg](1-q_c)\Bigg) \\
&+ \int_{\Theta_1}^{1} q^{-1} \\
&\Bigg(- \ell^2 E[M_{0,2}(1-\rho)\Omega_2^{-\ell}(1 + c(1-\tau))\frac{F(qX_1, B_0)}{\partial B_{j,0}}\mathcal{B}_2(q)^{\ell-1}B_{\$,0}(1+c)(1-q_c)] \\
&- \ell^2 E\Bigg[M_{0,2}(1 + c(1-\tau))^{\ell}\Bigg((1-\rho)\left[\Omega_2^{-\ell}\mathcal{E}_{j,2}(B_{\$,1}(q) + B_0)^{\ell}(1+c)\right](1-q_c)]\Bigg)\Bigg)dq \;+\; o(\varepsilon_1^{1/\ell})
\end{aligned}
$$

Now, the really surprising effect is that when $B_{\$,0}$ is close to zero, the integral produces a logarithm term through the following lemma.

Lemma D.1

$$
\int_{\Theta_1^* B}^{1} f(q)q^{-1}dq \;\approx\; -\log(\Theta_1^* B)f(\Theta_1^* B)
$$

as $B \to 0$.

Thus, the key term is

$$
\begin{aligned}
&E[(M_{0,1}c_1\mathcal{E}_{j,1} + M_{0,2}c\mathcal{E}_{j,2})](\tau - q_c(j)) + \ell^2(B_{\$,0})^{\ell}\log(B_{\$,0}\Theta_1^*) \\
&E\Bigg[M_{0,2}(1 + c(1-\tau))^{\ell}(1-\rho)\left[\Omega_2^{-\ell}\mathcal{E}_{j,2}(1+c)\right](1 - q_c(\$))\Bigg]
\end{aligned}
$$

which should be non-positive for $j \neq \$$ and zero for $j = \$$. This leads to the inequality

$$\begin{aligned}
\frac{q_c(\$) - q_c(j)}{\tau - q_c(\$)} \;&\leq\; \frac{E^{\$}[\Omega_2^{-\ell}\mathcal{E}_{j,2}]}{E^{\$}[\Omega_2^{-\ell}]}\frac{c_1 e^{-r_1(\$)} + c_2 e^{-r_2(\$)}}{c_1 e^{-r_1(j)} + c_2 e^{-r_2(j)}} - 1 \\
&= \frac{E^{\$}[\Omega_2^{-\ell}\mathcal{E}_{j,2}]}{E^{\$}[\Omega_2^{-\ell}]E^{\$}[\mathcal{E}_{j,2}]}\frac{c_1 e^{r_2(\$)-r_1(\$)} + c_2}{c_1 e^{r_2(j)-r_1(j)} + c_2} - 1 \\
&= \frac{\mathrm{Cov}^{\$}(\Omega_2^{-\ell}, \mathcal{E}_{j,2})}{E^{\$}[\Omega_2^{-\ell}]E^{\$}[\mathcal{E}_{j,2}]} \;+\; \frac{E^{\$}[\Omega_2^{-\ell}\mathcal{E}_{j,2}]}{E^{\$}[\Omega_2^{-\ell}]E^{\$}[\mathcal{E}_{j,2}]}\frac{c_1(e^{r_2(\$)-r_1(\$)} - e^{r_2(j)-r_1(j)})}{c_1 e^{r_2(j)-r_1(j)} + c_2}
\end{aligned}$$

where we have used that $E^{\$}[\mathcal{E}_{j,2}] \;=\; e^{r_2(\$)-r_2(j)}$.

Proposition D.2 *It is never optimal to buy back debt.*

Proof. We need to show that

$$\begin{aligned}
&-\sum_j E_1\left[M_{1,2}\left[\left(1 - (1-\rho)\left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,2}\right]\right]B_{j,1} \\
&+\; E_1\left[M_{1,2}\left[-\mathcal{B}_2(-B_1+B_0)\left(1 - \left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^{\ell}\right) + \Omega_2\ell(\ell+1)^{-1}\left(1 - \left(\frac{\mathcal{B}_2(-B_1+B_0)}{\Omega_2}\right)^{\ell+1}\right)\right]\right]
\end{aligned}$$

is monotone decreasing in $B_{j,1}$. Taking the derivative, we get

$$\begin{aligned}
& - E_1 \left[M_{1,2} \left[\left(1 - (1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell} \right) (1+c)\mathcal{E}_{j,2} \right] \right] \\
& - \sum_k E_1 \left[M_{1,2} \left[\left(\ell(1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell-1} \Omega_2^{-1} \right) (1+c)(1+c(1-\tau))\mathcal{E}_{j,2} \right] \right] B_{k,1} \\
& + (1+c(1-\tau)) E_1 \left[M_{1,2}\mathcal{E}_{j,2} \right] \\
& - E_1 \left[M_{1,2}(\ell+1) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell} (1+c(1-\tau))\mathcal{E}_{j,2} \right. \\
& \left. + \ell \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell} (1+c(1-\tau))\mathcal{E}_{j,2} \right] \\
= \; & -E_1 \left[M_{1,2} \left[\left(1 - (1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell} \right) (1+c)\mathcal{E}_{j,2} \right] \right] \\
& - \sum_k E_1 \left[M_{1,2} \left[\left(\ell(1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell-1} \Omega_2^{-1} \right) (1+c)(1+c(1-\tau))\mathcal{E}_{j,2} \right] \right] B_{k,1} \\
& + (1+c(1-\tau)) E_1 \left[M_{1,2}\mathcal{E}_{j,2} \right] \\
& - E_1 \left[M_{1,2} \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell} (1+c(1-\tau))\mathcal{E}_{j,2} \right]
\end{aligned}$$

By replacing c with $c(1-\tau)$, we get that it suffices to show that

$$\begin{aligned}
E_1 \left[M_{1,2}\mathcal{E}_{j,2} \right] \; \leq \; & E_1 \left[M_{1,2} \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell} \mathcal{E}_{j,2} \right] \\
& + E_1 \left[M_{1,2} \left[\left(1 - (1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell} \right) \mathcal{E}_{j,2} \right] \right] \\
& + \sum_k E_1 \left[M_{1,2} \left[\left(\ell(1-\rho) \left(\frac{\mathcal{B}_2(-B_1 + B_0)}{\Omega_2} \right)^{\ell-1} \Omega_2^{-1} \right) \mathcal{E}_{j,2} \right] \right] B_{k,1}
\end{aligned}$$

which holds trivially because the last term can in fact be dropped. Q.E.D.

E Additional results: Local currency and dollar debt

The main goal of our paper is to explain the dominance of the dollar compared to other major international currencies primarily with global firms in mind. While it is not our primary focus to explain why firms in emerging markets issue debt in dollars as opposed to local currency, the mechanisms underlined in our paper do yield some predictions about that as well. In this appendix, we take as given the dominance of dollar among the major global currencies, and we investigate whether debt view can be used to explain the mixture of dollar- and local-currency denominated debt for non-financial firms in a cross-section of emerging market economies.

E.1 Results

We develop and test the predictions of an extension of our model using a cross-section of the emerging market economies for which data on corporate debt in different currencies are available.[45] We prove the following extension of Theorem 1.1 for the case wherein firms issue a mixture of local currency (LC) and dollar-denominated debt (see Theorem E.2 in the Appendix for the proof. Note that, while Proposition E.1 is a partial equilibrium result, it still holds true in general equilibrium when debt overhang costs are sufficiently small).

Proposition E.1 *Suppose that (1) $q = q(\$)$ (that is, issuing in LC costs the same as issuing in dollars); (2) the variance of all shocks is sufficiently small; and (3) issuing debt in both LC and dollars is optimal; (4) ℓ is close to 1. Then,*

(a) the fraction $\frac{B_t}{B_t(\$)\mathcal{E}_{\$,i,t}}$ is monotone increasing in the covariance $\mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ if and only if $B_t \geq B_t(\$)\mathcal{E}_{\$,i,t}$;

[45] Data were obtained from the Institute for International Finance (IIF) for the period from 2005 Q1 to 2018 Q2. The countries in our sample are Argentina, Brazil, Chile, China, Colombia, Czechia, Hong Kong, Hungary, India, Indonesia, Israel, Republic of Korea, Malaysia, Mexico, Poland, Russian Federation, Saudi Arabia, Singapore, South Africa, Thailand and Turkey.

(b) the fraction $\frac{B_t}{B_t(\$)\mathcal{E}_{\$,i,t}}$ *is always monotone decreasing in* $\sigma_{i,\varepsilon}$.

The intuition for the first theoretical result is that local currency debt partly replicates insurance properties of the dominant currency in downturns, while it is a better hedge against domestic productivity shocks. The second result is that volatile inflation generates volatility of profits which the firms avoid by issuing less local currency debt.

Items (a)-(b) of Proposition E.1 directly translate into the testable empirical hypotheses. We test the two implications of our theory:

1. The local currency share of corporate debt is *higher* for countries in which domestic inflation correlates more with US inflation when controlling for relevant factors.

2. Firms in countries with more volatile domestic inflation tend to have less debt denominated in local currency.

Figure 12 shows the mean of the debt ratio, $\frac{L\bar{C}U}{USD}_i$, for each country in our sample. The left-hand panel shows several outliers: China and the EU countries in the sample (Czechia, Hungary, and Poland), while the right-hand panel shows the rest of the countries. We exclude outliers from our regressions and focus only on the sample of countries listed in the right-hand panel.

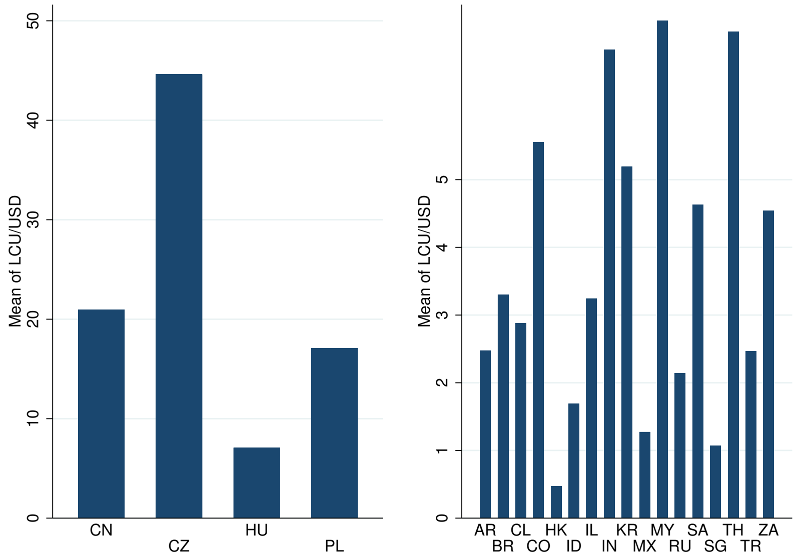
We find statistically significant evidence for the first prediction. Our second test results in a coefficient with the predicted sign, yet statistically insignificant.

In order to test the first hypothesis, we proceed as follows. For each in our sample, we estimated the following time series regression:

$$\pi_t^i = \gamma_0 + \gamma_1 \cdot Ret\_MSCIACWorld_t + \Gamma \cdot Ret\_DomesticStockIndex_t^i + \pi_t^{res,i}, \tag{7}$$

where π_t^i is the domestic monthly inflation rate in and $Ret\_MSCIACWorld_t$ is the monthly return on the MSCI AC World Index. $Ret\_DomesticStockIndex_t^i$ is the monthly return on

Figure 12: Mean of the local currency to USD debt ratio by country



Source: IIF, authors' calculations

the domestic stock market index. $\pi_t^{res,i}$ are the residuals from this regression. We also run the following regression for the US:

$$\pi_t^{US} = \mu_0 + \mu_1 Ret\_MSCIACWorld_t + \pi_t^{res,US}\,, \tag{8}$$

We then run the following regression to compute a proxy for the covariance $\mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ between the residual domestic inflation and residual US inflation (see item (a) of Proposition E.1),

$$\pi_t^{res,i} = \alpha + \beta \pi_t^{res,US} + \epsilon_t\,,$$

where $\pi_t^{res,i}$ is the residual domestic monthly inflation rate in from (7) and $\pi_t^{res,US}$ is the residual monthly inflation rate in the US from (8). We denote the estimated slope coefficient by $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$.

We then run the following cross-sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_1 + \beta_1 \hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}} + X_i + \eta_i\,. \tag{9}$$

Here, $\frac{L\bar{C}U}{USD}_i$ is the average ratio of debt denominated in local currency to debt denominated in dollars for corporates in the countries of the dataset; X_i denotes other control variables.

Item (a) of Proposition E.1 predicts that the coefficient β_1 in the regression (9) should be positive.

To test the second hypothesis, we calculate the standard deviation of $\pi_t^{res,i}$ as a proxy for $\sigma_{\varepsilon,i}$ in Proposition E.1, and then run the following cross-sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_2 + \beta_2 \sigma_i^{\pi_t^{res,i}} + X_i + \eta_i\,. \tag{10}$$

Proposition E.1, item (b) predicts that $\beta_2 < 0$.

In column (1), we run univariate regressions In column (2), we add an additional control variable $ka\bar{o}pen_i$: a financial openness index obtained from Chinn and Ito (2006). In column (3), we take the predictions of the model literally as they appear in item (a) of the Proposition E.1: $\beta_1 > 0$ for countries where $\frac{L\bar{C}U}{USD}_i > 1$ and we exclude Hong Kong where $\frac{L\bar{C}U}{USD}_i < 1$. In all three columns, regressions corroborate *Hypothesis CS-1.* [46] The first three columns are in line with the predictions of our theory. Column (4) of Table 10 shows the results of regression (10). Although the result is lacking statistical significance, the sign of the coefficient is indeed consistent with our theoretical prediction.

[46] All our results are qualitatively and quantitatively similar when we use raw domestic and US inflation rates, instead of residuals. Moreover, all results remain valid if we use the share of local currency debt in total debt instead of the ratio of local currency debt to dollar debt.

Table 10: The cross-section of the local currency to dollar debt ratio

	(1)	(2)	(3)	(4)
	$\frac{L\bar{C}U}{USD}_i$	$\frac{L\bar{C}U}{USD}_i$	$\frac{L\bar{C}U}{USD}_i$	$\frac{L\bar{C}U}{USD}_i$
$\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$	6.523***	6.094***	6.019***	
	(0.896)	(1.097)	(1.029)	
$ka\bar{o}pen_i$		-0.233	-0.192	-0.796*
		(0.347)	(0.428)	(0.398)
$\sigma_i^{\pi_t^{res,i}}$				-1.784
				(1.479)
Observations	17	17	16	17
R-squared	0.697	0.709	0.664	0.254

Notes: Robust standard errors in parentheses. *, **, *** denote significance at the 10, 5, and 1% levels respectively. $\frac{L\bar{C}U}{USD}_i$ is the mean share of local currency debt obtained from the IIF for each of the 17 emerging market economies between 2005 Q1 and 2019 Q4. $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$ is the estimated regression coefficient for a linear regression of residuals of monthly domestic inflation rate from (7) on the residuals of the US inflation rate from (8). $ka\bar{o}pen_i$ is the mean of the Chinn-Ito financial openness index for each country (average of the data available between 1970-2018). $\sigma_i^{\pi_t^{res,i}}$ is the standard deviation of the residuals of the monthly domestic inflation rate obtained from (7). In column (3), Hong Kong is excluded since the share of local currency debt to dollar debt is less than 1.

E.2 Proof of Proposition E.1

We first state the following extension of the Theorem 1.1 for the case of firms borrowing both in local currency and in dollars.

Theorem E.2 *Suppose that $q = q(\$)$. Then, issuing in a mixture of local currency and dollars is optimal if and only if*

$$\frac{\bar{q}(j,\$)}{\bar{q}(\$)} - 1 \ \leq \ \frac{\mathrm{Cov}_t^{\$}\left(\left(\frac{\Omega_{t+1}}{\mathcal{B}_{t+1}(B_t)}\right)^{-\ell}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[\left(\frac{\Omega_{t+1}}{\mathcal{B}_{t+1}(B_t)}\right)^{-\ell}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]}$$

for all $j = 1, \cdots, N$.

Proof of Theorem E.2 and Proposition E.1. The standard Kuhn-Tucker conditions that borrowing only in LC and dollars is optimal if and only if

$$\begin{aligned}
& E_t\left[M_{t,t+1}\left[\left(1-(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,t+1}\right]\right](1-q(j)) \\
& + E_t\left[M_{t,t+1}\left[\left(-\ell(1-\rho)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell-1}\Omega_{t+1}^{-1}\right)(1+c)\mathcal{E}_{j,t+1}\right]\mathcal{B}_{t+1}(B_t)\right](1-q(\$)) \\
& - (1+c(1-\tau))E_t\left[M_{t,t+1}\mathcal{E}_{j,t+1}\right] \\
& + E_t\left[M_{t,t+1}(\ell+1)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,t+1}\right. \\
& \left. - \ell\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,t+1}\right] \leq 0
\end{aligned}$$

for all j with the identity for $j = i, \$$. This inequality can be rewritten as

$$\bar{q}(j,\$)\frac{E_t[M_{t,t+1}\mathcal{E}_{j,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\mathcal{E}_{j,t+1}\right]} \ \leq \ 1 \ = \ \bar{q}(\$)\frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)^{\ell}\mathcal{E}_{\$,i,t+1}\right]}$$

and the first claim follows.

For the LC-\$ mixture, we assume for simplicity that $\ell = 1$. Then, we get the system

$$
\begin{aligned}
1 &= \bar{q}(\$)\frac{E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)\mathcal{E}_{\$,i,t+1}\right]} \\
1 &= \bar{q}(\$)\frac{E_t[M_{t,t+1}]}{E_t\left[M_{t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{t+1}}\right)\right]}
\end{aligned}
$$

whereby

$$
\mathcal{B}_{t+1}(B_t) = (1 + c(1-\tau))\,(B_t + B_t(\$)\mathcal{E}_{\$,i,t+1})
$$

Thus, we get the system

$$
\begin{aligned}
E_t[M_{t,t+1}\Omega_{t+1}^{-1}]B_t + E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t(\$) &= \tilde{q}(\$)E_t[M_{t,t+1}] \\
E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t + E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]B_t(\$) &= \tilde{q}(\$)E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}]
\end{aligned}
$$

where we have defined

$$
\tilde{q}(\$) = \bar{q}(\$)/(1 + c(1-\tau))\,.
$$

Thus,

$$
\begin{pmatrix} B_t \\ B_t(\$) \end{pmatrix} = \tilde{q}(\$)\Delta_t^{-1}\begin{pmatrix} E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2] & -E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}] \\ -E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}] & E_t[M_{t,t+1}\Omega_{t+1}^{-1}] \end{pmatrix}\begin{pmatrix} E_t[M_{t,t+1}] \\ E_t[M_{t,t+1}\mathcal{E}_{\$,i,t+1}] \end{pmatrix}.
$$

where

$$
\Delta_t = E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]E_t[M_{t,t+1}\Omega_{t+1}^{-1}] - (E_t[M_{t,t+1}\Omega_{t+1}^{-1}\mathcal{E}_{\$,i,t+1}])^2
$$

Thus,

$$\frac{B_t}{B_t(\$)\mathcal{E}_{t,\$,i}} \;=\; \frac{-\mathrm{Cov}_t^{\$}(\Omega_{t+1}^{-1}\mathcal{E}_{t,t+1,\$,i},\;\mathcal{E}_{t,t+1,\$,i}^{-1})}{\mathrm{Cov}_t^{\$}(\Omega_{t+1}^{-1},\;\mathcal{E}_{t,t+1,\$,i}^{-1})}\,.$$

Thus,

$$\frac{B_t}{B_t(\$)\mathcal{E}_{t,\$,i}} \;=\; \frac{-\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{\$,t,t+1}\right)^{-1},\mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}{\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{i,t,t+1}\right)^{-1},\mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}\,.$$

Let now $\tilde{a}_{i,t+1} \;\equiv\; \log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}})-\beta\tilde{a}_{\$,t+1}$ where $\tilde{a}_{\$,t+1}=\log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{\$,t+1}})$ and where β is such that $\tilde{a}_{i,t+1}$ and $\tilde{a}_{\$,t+1}$ are uncorrelated.

Recall also that we assume that

$$\log\mathcal{P}_{i,t,t+1} \;=\; -\hat{\alpha}_i\tilde{a}_{\$,t+1}-\alpha_i\tilde{a}_{i,t+1} \;+\; \varepsilon_{i,t+1},\;\log\mathcal{P}_{\$,t,t+1} \;=\; -\tilde{\alpha}_{\$}\hat{a}_{\$,t+1}+\varepsilon_{\$,t+1}$$

where $\varepsilon_{i,t+1}\sim N(0,\sigma_{\varepsilon,i}^2)$. We also allow $\sigma_{\varepsilon,i,\$} \;\equiv\; \mathrm{Cov}_t(\varepsilon_{i,t+1},\varepsilon_{\$,t+1}) \;\neq 0$. Then, to the first order in variance, the measure change is irrelevant and

$$\begin{aligned}
&-\;\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{\$,t,t+1}\right)^{-1},\mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)\\
&\approx\;-\mathrm{Cov}_t(-\tilde{a}_{i,t+1}-\beta\tilde{a}_{\$,t+1}\;+\alpha_{\$}\tilde{a}_{\$,t+1}\;-\;\varepsilon_{\$,t+1},-\alpha_{\$}\tilde{a}_{\$,t+1}\;+\;\varepsilon_{\$,t+1}+\alpha_i\tilde{a}_{i,t+1}+\hat{\alpha}_i\tilde{a}_{\$,t+1}\;-\;\varepsilon_{i,t+1})
\end{aligned}$$

whereas

$$\begin{aligned}
&\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{i,t,t+1}\right)^{-1},\mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)\\
&\approx\;\mathrm{Cov}_t(-\tilde{a}_{i,t+1}-\beta\tilde{a}_{\$,t+1}+\alpha_i\tilde{a}_{i,t+1}+\hat{\alpha}_i\tilde{a}_{\$,t+1}\;-\;\varepsilon_{i,t+1},\\
&-\alpha_{\$}\tilde{a}_{\$,t+1}\;+\;\varepsilon_{\$,t+1}+\alpha_i\tilde{a}_{i,t+1}+\hat{\alpha}_i\tilde{a}_{\$,t+1}\;-\;\varepsilon_{i,t+1})
\end{aligned}$$

In the small variance approximation, we that's get

$$\frac{B_t}{B_t(\$)\mathcal{E}_{t,\$,i}} \approx \frac{\sigma^2_{\varepsilon,\$} - \sigma_{\varepsilon,i,\$} + \alpha_i\sigma_c^2 + \alpha_\$^2\sigma_c^2(\$) - (\alpha_\$ + \alpha_i\alpha_\$)\sigma_c(i,\$)}{\sigma^2_{\varepsilon,i} - \sigma_{\varepsilon,i,\$} + (1-\alpha_i)(\alpha_\$\sigma_c(i,\$) - \alpha_i\sigma_c^2)}$$

where $\sigma_c^2 = \mathrm{Var}_t[\log(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}})]$ and $\sigma_c(i,\$) = \mathrm{Cov}_t[\log(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}}), \log(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{\$,t+1}})]$.
The claims (monotonicity in $\sigma_{\varepsilon,i,\$}$ and $\sigma^2_{\varepsilon,i}$) follow then by direct calculation. Q.E.D.